# 14 Distributions, transformations and spread

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

**KEY WORDS** standard deviation • frequency • median

## Objective

- Compare mean, median , range and standard deviation qualitatively
- Predict the effect of adding or multiplying every data value
- Read frequency information without confusing values and counts

Original public practice; marks below are classroom self-check points, not official criteria.

## 1 Key knowledge and worked examples

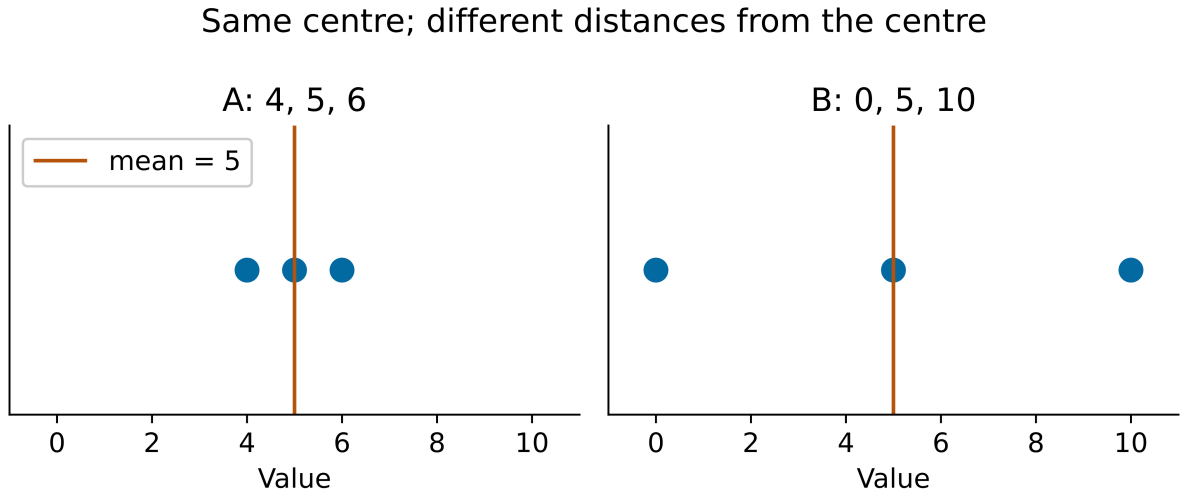
The mean uses the sum of all observations divided by their count; the median is the middle of the ordered observations. Frequency means a value repeats: if 2 occurs three times and 8 once, the mean is $(3 \cdot 2{+}8)/4{=}3.5$, not $(2{+}8)/2$. The horizontal positions on a display are values and the frequencies determine how many observations they represent.

The range is maximum minus minimum; standard deviation describes typical distances from the mean. You need not memorise a full formula to compare simple equally centred distributions: data clustered close to the mean have smaller standard deviation than data moved further away symmetrically. An outlier can alter mean and spread substantially while leaving the median stable.

Adding c to every observation raises both mean and median by c but leaves range and standard deviation unchanged, because pairwise distances do not change. Multiplying every observation by a positive k multiplies mean, median, range and standard deviation by k. More generally spread scales by |k|, while a negative multiplier reverses ordering.

When comparing plots, inspect the scale and the number of observations before judging apparent width. A graph stretched horizontally can look more variable without representing different values. A claim about equality of standard deviations needs the actual distributions, not merely equal ranges or equal means.

A frequency counts repeated observations. Values 1, 3, 7 with frequencies 2, 1, 1 represent four observations: 1, 1, 3, 7. $\bar{x} = \sum fx / \sum f = (2(1) + 1(3) + 1(7))/4 = 3$. The median is $(1 + 3)/2 = 2$; it need not equal the mean.




*Same mean, but B's values lie further from the centre, so B has the larger standard deviation*

**Worked example.** A={4,5,6} and B={0,5,10} both have mean and median 5, but B has greater spread. Adding 7 gives A+7={11,12,13}, mean 12 and the same standard deviation as A. Multiplying A by 2 gives {8,10,12}; its mean is 10 and its range and standard deviation double. Equal range alone does not guarantee equal standard deviation.

## 2 Practice

**2.1** Add 10 to every observation. Standard deviation: [1]

- **A** Stays the same
- **B** Increases by 10
- **C** Doubles

**2.2** 2 occurs three times; 6 occurs once. Mean: [1]

- **A** 3
- **B** 4
- **C** 8

**2.3** The range of 3, 7, 9 is _____. [1]

**2.4** Two distributions with equal means must have equal spread. [1]

## 3 Exam-style questions

Use the data stated in each original task.

**3.1** Values 2, 5, 8 have frequencies 3, 2, 1. Find mean, median and range. [3]

**3.2** Every observation above is changed by $z = 3x - 4$. Find the new mean, median and range. State how standard deviation changes without computing it. [3]

**3.3** Compare A = {0, 0, 10, 10} and B = {0, 5, 5, 10}. Both have range 10. Must their standard deviations agree? Justify. [3]

## Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

**2.1 A** – All deviations from the shifted mean are unchanged. B treats a translation as added spread; C treats it as multiplication. Distances from the mean change in neither way.

Correct response linked to the given case.

**2.2 A** – (2+2+2+6)/4=3. B averages the distinct values without their frequencies; C adds those values without dividing by the observation count.

Correct response linked to the given case.

**2.3** 6 – Maximum minus minimum is 9-3.

Correct response linked to the given case.

**2.4** False – Centre and spread describe different features.

Correct response linked to the given case.

**3.1** Data are 2,2,2,5,5,8. $\bar{x} = (3(2) + 2(5) + 1(8))/6 = 4$. Median is the average of positions 3 and 4, $(2 + 5)/2 = 3.5$. Range $R = 8 - 2 = 6$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

**3.2** Mean $\bar{z} = 3\bar{x} - 4 = 3(4) - 4 = 8$. Median is $3(3.5) - 4 = 6.5$. Range is $3(6) = 18$. Standard deviation triples; subtracting 4 changes no distance from the mean.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

**3.3** Both means are 5. Every A observation is 5 from its mean; B distances are 5,0,0,5. B therefore has smaller mean squared deviation and smaller standard deviation. Equal range alone is insufficient.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.