13 Ratios, unit conversion and successive percentages

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS unit rate • rate

Objective

- Use proportional relationships while preserving units
- Convert rates and distinguish direct from inverse relationships
- Calculate successive changes and percentage comparisons using the correct base

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

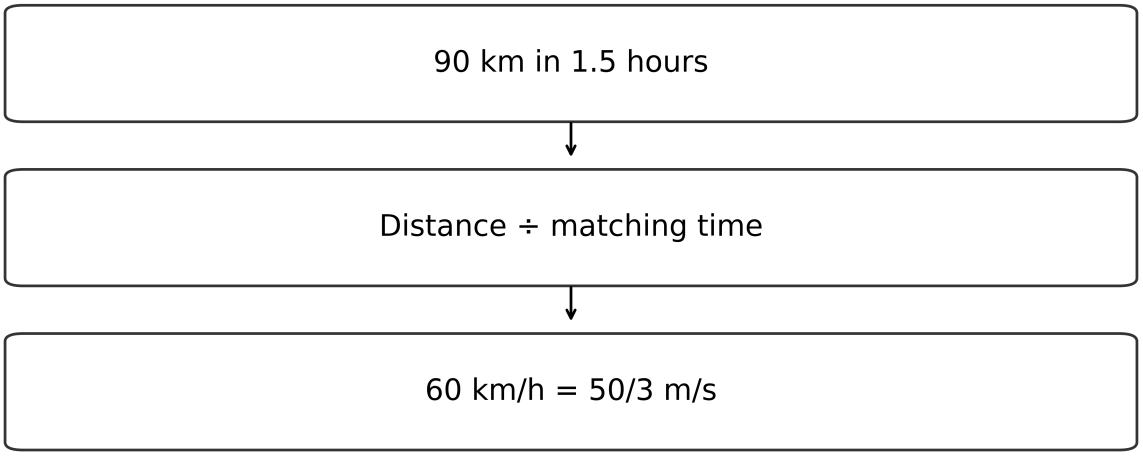
A ratio compares quantities in a chosen order. If red:blue=3:5 and the total is 64, the eight equal parts each contain eight, so red is 24 and blue 40. The ratio 3/5 describes red relative to blue; 3/8 describes red relative to the total. Read the requested denominator rather than using the first visible fraction.

A unit rate divides quantity by its corresponding time, distance or count. Write conversion factors so unwanted units cancel: 72 km/hour times 1000 m/km divided by 3600 s/hour is 20 m/s. Area and volume conversions square and cube the length factor. A one-centimetre square is not one metre squared divided by one hundred.

In direct proportion y=kx, doubling x doubles y; in inverse proportion xy=k, doubling x halves y. Constant work shared by identical workers gives an inverse time model only when productivity and conditions are fixed. A fixed initial fee added to a rate makes an affine linear model, not direct proportion through the origin.

A percentage change divides the change by the original comparison base. Successive percentage changes multiply their factors: a 20% discount followed by 20% increase gives $0.8 \cdot 1.2 = 0.96$ of the original. Distinguish percentage points from relative percent: 30% to 36% is six points, but a 20% relative increase.

A rate compares quantities with their units. $v = d/t = (90\,\text{km})/(1.5\,\text{h}) = 60\,\text{km/h}$. Convert using cancelling units: $v = (60\,\text{km/h})(1000\,\text{m/km})/(3600\,\text{s/h}) = 50/3\,\text{m/s}$. Do not convert only the numerator of a compound rate.



Worked steps: 90 km in 1.5 hours → Distance ÷ matching time → 60 km/h = 50/3 m/s

Worked example. A 250-unit price discounted 20% becomes 200; increasing that result 20% gives 240, four percent below 250. A pump delivers 18 litres in 3 minutes, so its rate is 6 L/min or 0.1 L/s. At a fixed rate, 45 litres takes 7.5 minutes. If a share rises from 40% to 50%, the rise is 10 percentage points and 25% relative to the old share.

2 Practice

2.1 A ratio A:B=2:3 with total 45 gives A: [1]

- **A** 18
- **B** 30
- **C** 22.5

2.2 A price rises from 80 to 100. Its percentage increase is: [1]

- **A** 25%
- **B** 20%
- **C** 125%

2.3 36 km/hour equals _____ m/s. [1]

2.4 y=4x+7 is directly proportional to x. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Mix red and blue paint in ratio 4:7 to make 33 litres. Find each amount. Red paint costs 18 yuan per litre and blue 12. Find total cost. [3]

3.2 A 400-yuan item falls 15% then rises 10%. Find the final price and the single overall percentage change. [3]

3.3 A pump supplies 0.15 litres per second. How long does 54 litres take? If a square has side 30 cm, give its area in square metres. [3]

Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

2.1 **A** – Two of five equal parts gives $45 \cdot 2/5{=}18$. B uses two thirds of the total rather than two fifths; C assumes equal shares despite the 2:3 ratio.

Correct response linked to the given case.

2.2 **A** – The change 20 is divided by original 80. B divides by final price; C is final price as a percentage of original, not percentage increase.

Correct response linked to the given case.

2.3 10 – $36 \cdot 1000/3600{=}10$.

Correct response linked to the given case.

2.4 False – The nonzero intercept prevents y/x being constant.

Correct response linked to the given case.

3.1 There are 11 parts, so each is 3 L. Red volume $V_r = 33(4/11) = 12\,\mathrm{L}$; blue volume $V_b = 33(7/11) = 21\,\mathrm{L}$. Cost $C = p_r V_r + p_b V_b = (18\,\text{yuan/L})(12\,\mathrm{L}) + (12\,\text{yuan/L})(21\,\mathrm{L}) = 468\,\text{yuan}$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence . Accurate conditions, checks or qualification.

3.2 Use successive multipliers. $P = P_0(1 - 0.15)(1 + 0.10) = (400\,\text{yuan})(0.85)(1.10) = 374\,\text{yuan}$. Relative change $r = (P - P_0)/P_0 = (374 - 400)/400 = -0.065$, a 6.5% decrease.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.3 $t = V/q = (54\,\mathrm{L})/(0.15\,\mathrm{L/s}) = 360\,\mathrm{s} = 6\,\mathrm{min}$. Convert side before squaring: $A = s^2 = (0.30\,\mathrm{m})^2 = 0.09\,\mathrm{m}^2$; the area conversion factor is 10000, not 100.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.