

12 Nonlinear functions, transformations and growth

Name: _____ Class: _____ Date: _____

Total: 13 marks**KEY WORDS** transformation • vertex form • parabola • rate

Objective

- Interpret zeros, vertices and parameters from useful function forms
- Distinguish additive linear change from multiplicative exponential change
- Transform a graph while tracking its input and output

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

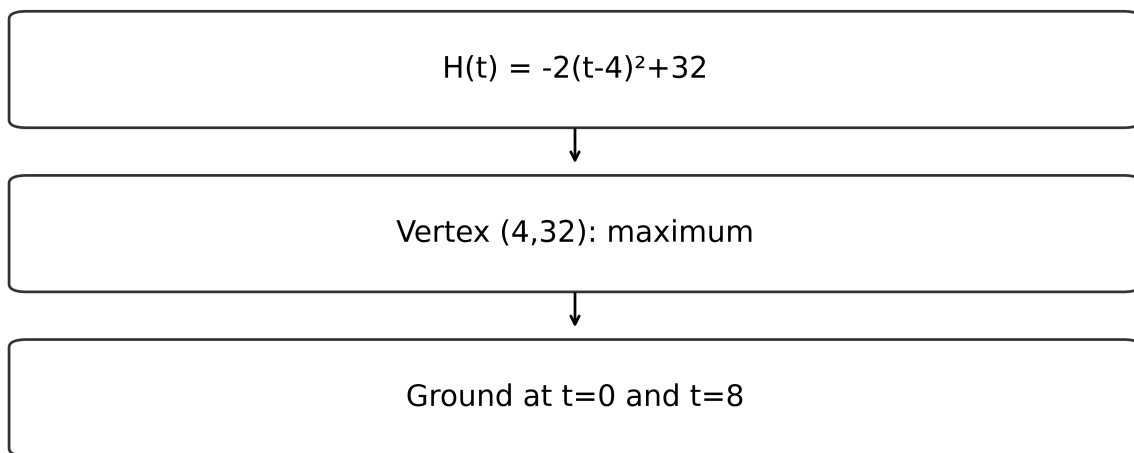
Vertex form $a(x-h)^2+k$ identifies the vertex (h,k) and opening direction; factored form identifies zeros. The same function can have both forms. In context, determine which features are meaningful: a maximum height occurs at the vertex of a downward parabola, while a zero can represent returning to ground level.

For $y=A b^t$, A is the value at $t=0$ and b is the factor per unit time. A growth rate r gives $b=1+r$; decay gives $b=1-r$. Equal differences indicate a linear model, while equal ratios indicate an exponential model for equal input steps. Neither model should be extrapolated beyond a justified domain without a contextual reason.

An output shift $f(x)+k$ moves every point vertically by k . An input shift $f(x-h)$ moves the graph right by h : the same old input is reached when the new x is larger. A multiplier outside scales output; a negative outside reflects across the x -axis. Determine the transformation algebraically instead of memorising a direction without checking a point.

For an exponential factor given over several time units, distinguish that interval from one unit. Doubling every three hours means $2^{(t/3)}$, not 2^t . For a quadratic, use the requested representation to read an extremum or solve for an input. Keep initial values, growth factors and growth percentages separate.

Vertex form $a(x-h)^2+k$ reveals the turning point (h,k) . For $H(t) = -2(t-4)^2 + 32$, the maximum is 32 when $t = 4$. Setting $H(t) = 0$ gives $(t-4)^2 = 16$, hence $t = 0, 8$. Restrict time to the physical interval when the problem represents a flight.



Worked steps: $H(t) = -2(t-4)^2 + 32 \rightarrow \text{Vertex } (4,32): \text{maximum} \rightarrow \text{Ground at } t=0 \text{ and } t=8$

Worked example. $h(t) = -4(t-3)^2 + 36$ has a maximum 36 at $t=3$, with zeros $t=0$ and 6. $N(t) = 80(1.25)^t$ starts at 80 and grows 25% per unit; $N(2) = 125$. A quantity doubling every three hours is $80 \cdot 2^{(t/3)}$, so after six hours it is 320. If $f(x) = x^2$, $f(x-2) + 5$ has vertex (2,5).

2 Practice

2.1 In $200(0.8)^t$, the decay rate per unit is: [1]

- A 20%
- B 80%
- C 200%

2.2 The vertex of $(x+3)^2 - 7$ is: [1]

- A (-3,-7)
- B (3,-7)
- C (-3,7)

2.3 100 grows by 10% twice. The result is _____. [1]

2.4 Equal additive differences prove exponential growth. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 A model height is $H(t) = -3(t - 2)^2 + 12$ metres, with t seconds from launch until return to ground. Find the domain, greatest height, and time interval with height at least 9 metres. [3]

3.2 A culture model starts with 120 cells and grows by a factor of 1.5 every two hours. Write $N(t)$ for hours t , find $N(4)$, and distinguish factor from percentage growth. [3]

3.3 If $f(x) = (x + 1)^2$, find the vertex of $g(x) = -2f(x - 3) + 5$ and its maximum or minimum. [3]

Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

2.1 **A** – $1 - 0.8 = 0.2$. **B** is the remaining fraction, not the decrease; **C** confuses initial quantity with a percentage rate.

Correct response linked to the given case.

2.2 **A** – $x + 3 = x - (-3)$, so $h = -3$ and $k = -7$. **B** reverses the horizontal shift; **C** reverses the vertical coordinate.

Correct response linked to the given case.

2.3 $121 = 100 \cdot 1.1^2 = 121$.

Correct response linked to the given case.

2.4 False – Constant differences indicate a linear relationship; equal ratios support exponential growth.

Correct response linked to the given case.

3.1 Ground means $H = 0$, so $(t - 2)^2 = 4$ and $t = 0, 4$. Domain is $0 \leq t \leq 4$ s. The negative square coefficient gives maximum $H(2) = 12$ m. For $H \geq 9$ m, $(t - 2)^2 \leq 1$, so $1 \leq t \leq 3$ s.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence
. Accurate conditions, checks or qualification.

3.2 $N(t) = 120(1.5)^{t/2}$ cells. $N(4) = 120(1.5)^2 = 270$ cells. The growth is 50% per two hours, not 150%; the four-hour factor is 2.25. The hourly factor would be $\sqrt{1.5}$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.

3.3 Substitute the input before interpreting signs. $g(x) = -2(x - 2)^2 + 5$ has vertex $(2, 5)$ and maximum 5. The old vertex at -1 moved three units right to 2.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.