

11 Nonlinear equations and mixed systems

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS candidate root • intersection • parabola

Objective

- Solve quadratic, absolute-value, rational and radical equations
- Reject candidate roots violating the original equation or domain
- Find intersections of a line and a nonlinear curve

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

Use structure before expanding. A factored quadratic equals zero when at least one factor is zero. An absolute-value equation $|u|=a$ has no real solution for $a<0$, one condition $u=0$ when $a=0$ and two conditions $u=\pm a$ when $a>0$. Account for all allowed cases, not just the positive branch.

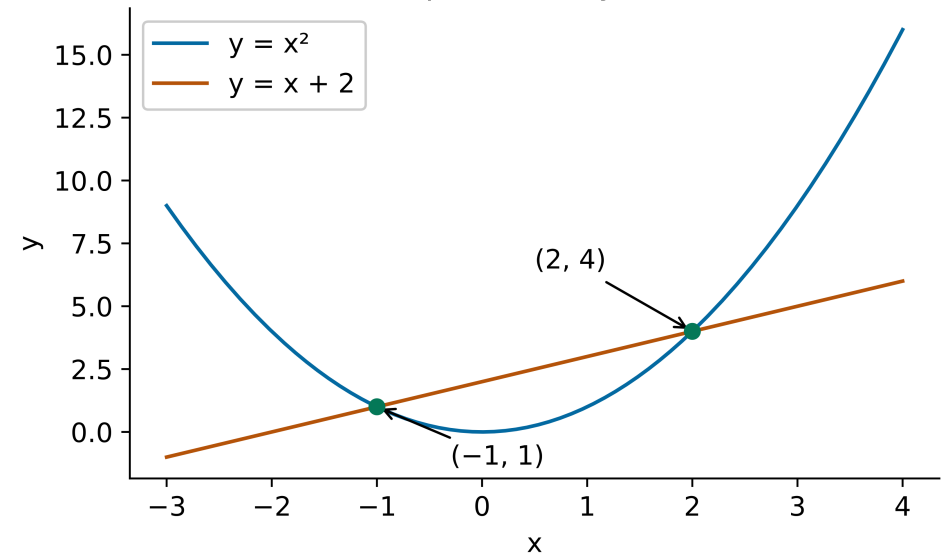
For a rational equation, exclude denominator zeros before multiplying through. Multiplication may create a candidate at an excluded value. For a radical equation, note that a principal square root is nonnegative, isolate it and square carefully. Squaring is a one-way implication until every resulting candidate is checked in the original.

To solve a line–parabola system, substitute the line expression into the quadratic. The real roots give x-values; use the line to recover each y. No real root means no real intersection. A repeated quadratic root gives tangency, while two different real roots give two intersections.

A graph or calculator can help locate candidates and check scale, but rounded intersections are not exact proof when the question asks an exact value or solution count. Relate graphical behaviour to the algebraic equation. A double root should not be counted twice as two different ordered pairs.

A candidate root must satisfy the original equation. In $\sqrt{x+6} = x$, the right side must be nonnegative. Squaring gives $x^2 - x - 6 = 0$, so candidates are 3, -2. Only 3 survives the original check: $\sqrt{9} = 3$.

Two ordered pairs satisfy both relations



A line can meet a parabola twice; both ordered pairs satisfy both relations

Worked example. $\sqrt{x+2}=x$ requires $x \geq 0$. Squaring yields $x^2 - x - 2 = 0$, so $x=2$ or -1 ; only 2 works in the original. For $y=x+2$ and $y=x^2$, solve $x^2 - x - 2 = 0$: $x=2, -1$, giving $(2, 4)$ and $(-1, 1)$. For $|2x-1|=5$, solve $2x-1=5$ or -5 to obtain $x=3$ or -2 .

2 Practice

2.1 Which solves $\sqrt{x+6}=x$ with real x ? [1]

- A 3
- B -2
- C Both

2.2 How many real solutions has $|x+1|=-2$? [1]

- A 0
- B 1
- C 2

2.3 For $(x-5)(x+1)=0$, the negative solution is _____. [1]

2.4 A repeated root gives two distinct intersections. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Solve $\sqrt{2x+3} = x$ over the real numbers, showing why any candidate is rejected. [3]

3.2 Solve simultaneously $y = x + 2$ and $y = (x - 1)^2$. Give exact ordered pairs and verify the number of intersections. [3]

3.3 Solve $|3x + 2| = 7$ and $(x^2 - 9)/(x - 3) = 6$. Explain why these equations have different solution counts. [3]

Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

2.1 A – Squaring gives $x=3$ or -2 ; only 3 makes both original sides nonnegative and equal. B fails substitution because $\sqrt{4}=2$, not -2 ; C retains that extraneous candidate.

Correct response linked to the given case.

2.2 A – Absolute value cannot be negative. B and C both assume a negative absolute value can occur; the nonnegative range excludes every candidate.

Correct response linked to the given case.

2.3 -1 – The second factor is zero at -1 .

Correct response linked to the given case.

2.4 False – Multiplicity does not create distinct ordered pairs.

Correct response linked to the given case.

3.1 Require $x \geq 0$. Squaring yields $x^2 - 2x - 3 = (x - 3)(x + 1) = 0$. Candidates are 3 and -1 . The original gives $\sqrt{9} = 3$ at 3, but $\sqrt{1} = 1 \neq -1$ at -1 . Only $x = 3$ works.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence
. Accurate conditions, checks or qualification.

3.2 Equate outputs to get $x + 2 = x^2 - 2x + 1$, or $x^2 - 3x - 1 = 0$. Thus $x = (3 \pm \sqrt{13})/2$ and $y = (7 \pm \sqrt{13})/2$ with matching signs. The discriminant 13 is positive, so there are two distinct real intersections; both expressions for y agree by the derived equation.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.

3.3 The absolute value gives $3x + 2 = 7$ or -7 , hence $x = 5/3$ or -3 . The rational equation requires $x \neq 3$ and simplifies to $x + 3 = 6$, whose sole candidate 3 is excluded. It has no solution.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.