10 Equivalent expressions and domain restrictions

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS equivalent expressions • factor

Objective

- Factor and expand expressions without changing their values
- Simplify rational and radical expressions on the original domain
- Use expression structure to identify zeros and coefficients

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

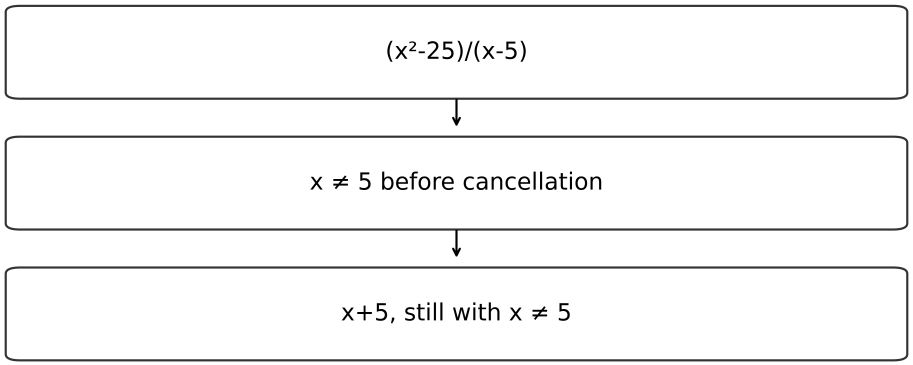
Expansion distributes each factor across the others; factorisation reverses that process. $x^2-5x+6=(x-2)(x-3)$, so the product form exposes zeros while the expanded form exposes coefficients. Choose the form that serves the question rather than treating one representation as always preferable.

Cancel factors, never terms in a sum. $(x^2-4)/(x-2)$ becomes $x+2$ only for $x\neq 2$, because the original expression is undefined at 2. A restriction may remain after every denominator disappears from the simplified formula. State it before cancelling, then carry it through comparisons or equation solving.

Exponent rules depend on their setting: $a^m a^n=a^{(m+n)}$ and $(a^m)^n=a^{(mn)}$ in the permitted real domain. Negative exponents form reciprocals and require nonzero bases. With real square roots, $\sqrt{(x^2)}=|x|$, because the principal square root is nonnegative; it is not x for every real x.

To check equivalence, a numerical substitution can expose a mistake, but one matching value does not prove two expressions equal. A valid algebraic identity plus matching domain does. When the question requests a coefficient, expand only as far as needed and account for every contribution to that power.

Equivalent expressions must agree on the stated domain . $E(x) = (x^2 - 25)/(x-5) = (x+5)$ only for $x \neq 5$. Cancellation removes a factor, but it cannot restore an excluded input. Also $\sqrt{x^2} = |x|$, because the principal square root cannot be negative.



Worked steps: $(x^2-25)/(x-5) \rightarrow x \neq 5$ before cancellation $\rightarrow x+5$, still with $x \neq 5$

Worked example. $(x^2-9)/(x-3)=(x-3)(x+3)/(x-3)=x+3$ for $x\neq 3$. At $x=3$ the original is undefined. For $x=-5$, $\sqrt{(x^2)}=5=|x|$. In $(2x+1)(x-4)$, the x coefficient is $-8+1=-7$ and the full expression is $2x^2-7x-4$.

2 Practice

2.1 Simplify $(x^2-16)/(x-4)$, with $x\neq 4$. [1]

- **A** $x+4$
- **B** $x-4$
- **C** x^2-4

2.2 For $x=-6$, $\sqrt{(x^2)}$ equals: [1]

- **A** 6
- **B** -6
- **C** 36

2.3 In $(x+2)(x+5)$, the x coefficient is _____. [1]

2.4 Matching at x=1 proves two expressions equivalent for all x. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Simplify $F(x) = (x^2 - 4x)/(x^2 - 16)$. State every original restriction and explain why F is not identical to $x/(x + 4)$ on that latter formula's whole domain. [3]

3.2 Find the coefficient of x in $(3x - 2)(x + 5)$ and evaluate $\sqrt{(x - 5)^2}$ when $x = 1$. [3]

3.3 A student tests $x = 2$ and concludes that x^2 and $2x$ are equivalent. Refute the conclusion and solve where they actually agree. [3]

Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

2.1 A – Difference of squares gives (x-4)(x+4). B loses the other factor x+4; C does not result from cancellation of whole factors.

Correct response linked to the given case.

2.2 A – The principal root is |x|. B violates the nonnegative principal-root convention; C is the squared value rather than its root.

Correct response linked to the given case.

2.3 7 – The two x terms contribute 5x+2x.

Correct response linked to the given case.

2.4 False – Different expressions can share one value.

Correct response linked to the given case.

3.1 Factor numerator $x(x-4)$ and denominator $(x-4)(x+4)$. The original domain excludes $x = 4, -4$. Cancellation gives $F(x) = x/(x+4)$ for $x \neq 4, -4$. The latter formula alone permits 4, where it gives $1/2$, while $F(4)$ is undefined.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence . Accurate conditions, checks or qualification.

3.2 Expansion gives $3x^2 + 15x - 2x - 10 = 3x^2 + 13x - 10$, so the coefficient is 13. The root is $|x-5|$; at $x = 1$, it is $|-4| = 4$, not -4 or 16.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.3 At $x = 3$ the outputs are 9 and 6, so one matching test is not proof. Solve $x^2 = 2x$ as $x(x-2) = 0$, giving only $x = 0, 2$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.