

Algebra

9260 unit 3.2 full package: Core A1-A26 with Extension additions, specimen-family anchors, unsampled lines labelled

1. Unit 3.2 at a glance

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Core A1-A26: notation through graphs, equations, inequalities and sequences.

Extension additions: subject-twice rearrangement, triple expansions, all quadratic factorising, fractional powers, quadratic fractions, formal proof, the full functions toolkit, line equations, exponential and trig graphs, differentiation with tangents and stationary points, completing the square, variation, all simultaneity, quadratic inequalities, quadratic sequences.

Must-know: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $a(x + b)^2 + c$ reads the turning point; $y - y_1 = m(x - x_1)$; $\frac{d}{dx}(kx^n) = knx^{(n-1)}$; the tangent's gradient IS the gradient at the point.

2. Formulae and subjects

Formulae and subjects

Substitute into expressions and formulae; rearrange by balanced operations.

Subject twice (Extension): $ax + 3 = 5xt - 2a$ gives $ax + 2a = 5xt - 3$, so $a(x + 2) = 5xt - 3$ and $a = (5xt - 3)/(x + 2)$. Collect the subject's terms before factorising; check numerically.

3. Expanding, factorising, fractions

Expanding, factorising, fractions

$(x + 2)(x - 3)(x + 1) = x^3 - 7x - 6$ (two first, then the third). $2x^2 - 7x - 15 = (2x + 3)(x - 5)$; $9x^2 - 25 = (3x - 5)(3x + 5)$.

Quadratic fractions (Extension): factor everything, cancel whole factors - the 2E specimen answers $(11m - 14)/(m^2 - 3m - 4)$ and $(2x + 3)/(x + 4)$ come from this machinery.

4. Algebraic proof

Algebraic proof

State the algebra, simplify, compare with the claim. $(2n + 1)^2 - (2n - 1)^2 = 8n$ - a multiple of 8 for every integer n .

The 2E specimen proves the $n^2 + n + 9$ family. Numeric examples are never a proof; the conclusion is stated in words.

5. Functions

Functions

$f(x)$ maps each input to one output; domain and range stated. Composite $fg(x) = f(g(x))$ - g first: the 2E specimen builds $fg(x) = (x - 5)^2 + 1$ and solves $fg(x) = 5$ to $x = 3$ (or 7).

Inverse by swap-and-rearrange: the 1E specimen's $f^{-1}(x) = x/3$ comes from $f(x) = 3x$. Check $f(f^{-1}(x)) = x$.

6. Lines and curves

Lines and curves

Lines: $m = (y_2 - y_1)/(x_2 - x_1)$; $y - y_1 = m(x - x_1)$; through (1, 7) and (2, 9): $y = 2x + 5$. Perpendicular gradients multiply to -1 (not sampled).

Curves: linear, quadratic, cubic, reciprocal; exponential $y = k^x$ and trig graphs (not sampled); turning points by completing the square: $x^2 - 6x + 5 = (x - 3)^2 - 4$, minimum (3, -4). Gradients and areas estimated from graphs.

7. Curve families and turning points

Curve families and turning points

Sketch and interpret linear, quadratic, cubic and reciprocal graphs; Extension adds exponential $y = k^x$ and the trigonometric graphs in degrees (specification lines, not sampled by the specimens).

Roots, intercepts and turning points read from sketches; turning points DEDUCED by completing the square: $x^2 - 6x + 5 = (x - 3)^2 - 4$ has its minimum at (3, -4).
Gradients and areas under graphs estimated by tangents and trapezia.

8. Differentiation

Differentiation

$d/dx(kx^n) = knx^{(n-1)}$, term by term. The gradient at a point is the tangent's gradient: $y = x^2$ at $x = 2$ has tangent $y = 4x - 4$; the 2E specimen's family is $dy/dx = 15$ with tangent $y = 15x - 36$.

Stationary points solve $dy/dx = 0$, classified as maxima, minima or points of inflection: $y = x^3 - 12x + 5$ has maximum $(-2, 21)$, minimum $(2, -11)$, and tangent $y = 15x - 49$ at $x = 3$.

9. Equations and inequalities

Equations and inequalities

Quadratics by factorising, completing the square, or the formula: $x^2 + 6x + 2 = 0$ gives $x = -3 \pm \sqrt{7}$ (the 1E specimen family). Pairs: two linear by elimination; linear with quadratic by substitution, keeping both points.

Inequalities: linear in one or two variables and quadratic - $x^2 - x - 6 > 0$ gives $x < -2$ or $x > 3$ (outside the roots). Variation: direct $y = kx^n$, inverse $y = k/x^n$ (not sampled).

10. Sequences

Sequences

Term-to-term and position-to-term rules; triangular, square, cube sequences.

Quadratic sequences (Extension): constant second difference, $a = \text{half of it}$.

5, 9, 17, 29, 45: second difference 4 so $a = 2$; subtract $2n^2$ and the residual $-2n + 5$ gives $2n^2 - 2n + 5$. Check on two positions before printing.

Check yourself

Check yourself

Substitution and subject-twice rearrangement; triple expansions; all factorising; quadratic fractions; formal proof. Functions with domain, range, composite and inverse.

Line equations; curve families sketched; completing the square; differentiation with tangents and stationary points; all solving routes and inequality types; linear and quadratic n th terms.