Course Companion

OxfordAQA · International GCSE · Mathematics

Every topic handout in one volume

全部专题讲义合集

exercises.lol

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Number

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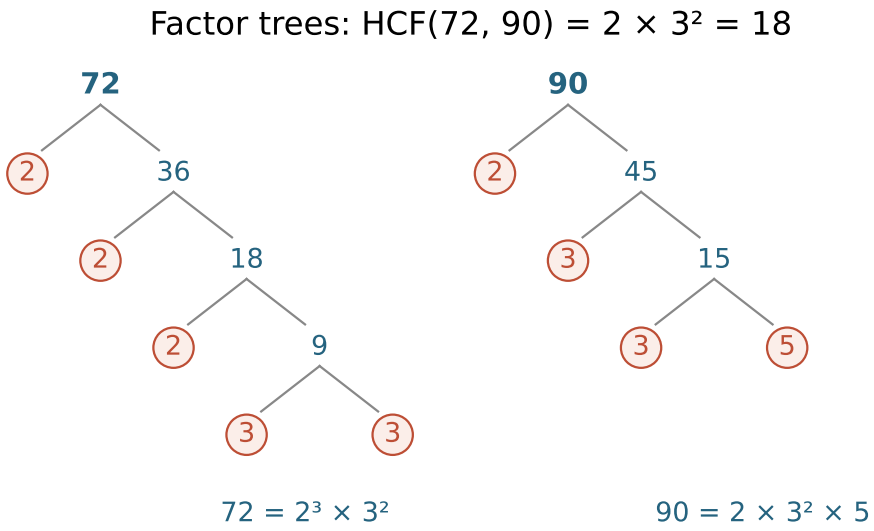
OxfordAQA International GCSE Mathematics (9260) unit 3.1 (specification for first assessment 2018). Core rows N1–N20 run from ordering and the four operations through factors and indices, **standard form** 标准形式, sets, rounding, FDP operators, percentage change and **ratio** 比; the **Extension additions** add index-form factorisation, fractional powers, exact surd work, bounds calculations, recurring decimals, reverse percentages, the compound interest formula and exponential growth and decay. The 2016 specimen pair (1E/2E, Extension tier) samples this unit at HCF, surds, fraction chains, bounds and exponential models; lines the specimens never sample are taught here as specification originals and labelled so.

Must-know for unit 3.1 (NO formulae sheet in 9260 – the recall list is the specification itself):

Family	Formulae
compound interest	$A = P\left(1 + \frac{r}{100}\right)^n$ (Extension)
bounds	value to the nearest unit u: LB $= v - \frac{u}{2}$, UB $= v + \frac{u}{2}$
recurring decimals	$x = 0.\overline{45} \Rightarrow 100x = 45.\overline{45} \Rightarrow x = \frac{45}{99} = \frac{5}{11}$
exponential models	growth $y = a\,b^t$ with $b > 1$; decay $0 < b < 1$ (Extension)

Ordering and the four operations

Order positive and negative **integers** 整数, decimals and fractions on the **number line** 数轴 with $=$, $\neq$, $<$, $>$ – convert to one form first when comparing ($\frac{3}{5} = 0.6$). The four operations with formal written methods cover integers, decimals, simple fractions proper and improper, and **mixed numbers** 带分数: $2\frac{1}{3} + 1\frac{1}{4} = \frac{28}{12} + \frac{15}{12} = \frac{43}{12} = 3\frac{7}{12}$. Inverse operations cancel and simplify: multiplication undoes division, subtraction undoes addition – use cancellation early ($\frac{15}{28} \times \frac{7}{9} = \frac{15}{4} \times \frac{1}{9} = \frac{5}{12}$ after cancelling the 7 and 3).



Factor trees: $72 = 2^3 \times 3^2$ and $90 = 2 \times 3^2 \times 5$, so HCF $= 2 \times 3^2 = 18$

Factors, primes, HCF and LCM

Even, odd, **prime** 质数, factor, **multiple** 倍数, common factor, common multiple – with the **highest common factor** 最大公因数 and **lowest common multiple** 最小公倍数 found from **prime factorisations** 质因数分解. The Extension addition writes every factor in **index form** 指数形式: $504 = 2^3 \times 3^2 \times 7$ and $588 = 2^2 \times 3 \times 7^2$, so the HCF is $2^2 \times 3 \times 7 = 84$ (the 1E specimen's answer) and the LCM is $2^3 \times 3^2 \times 7^2 = 3528$.

Indices, roots and surds

Positive integer powers with square and cube roots ($\sqrt{144} = 12$, $\sqrt[3]{125} = 5$); the index laws multiply and divide by adding and subtracting indices. The Extension addition brings **fractional powers** 分数指数 ($27^{1/3} = 3$, $8^{2/3} = 4$: cube-root then square) and **exact calculation with surds** 根式 – simplify by extracting squares and rationalise denominators with conjugates:

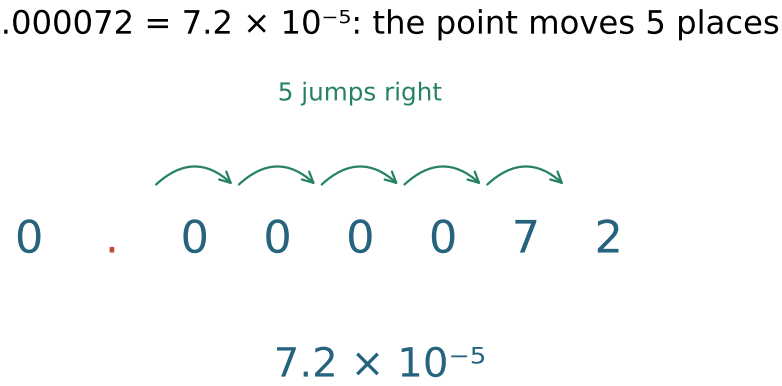
$$\sqrt{50} - \sqrt{18} + \sqrt{8} = 5\sqrt{2} - 3\sqrt{2} + 2\sqrt{2} = 4\sqrt{2}, \qquad \frac{5}{3-\sqrt{2}} = \frac{5(3+\sqrt{2})}{9-2} = \frac{5(3+\sqrt{2})}{7}$$

The 1E specimen's surd answer $4+4\sqrt{2}$ comes from this same machinery (family-level citation).

> **Worked example: a negative fractional power.** Evaluate $\left(\frac{16}{81}\right)^{-3/4}$. Deal with the three parts of the power one at a time. The minus sign flips the fraction; the **denominator** 分母 4 takes a fourth root; the **numerator** 分子 3 cubes:

$$\left(\frac{16}{81}\right)^{-3/4} = \left(\frac{81}{16}\right)^{3/4} = \left(\frac{3}{2}\right)^{3} = \frac{27}{8}.$$

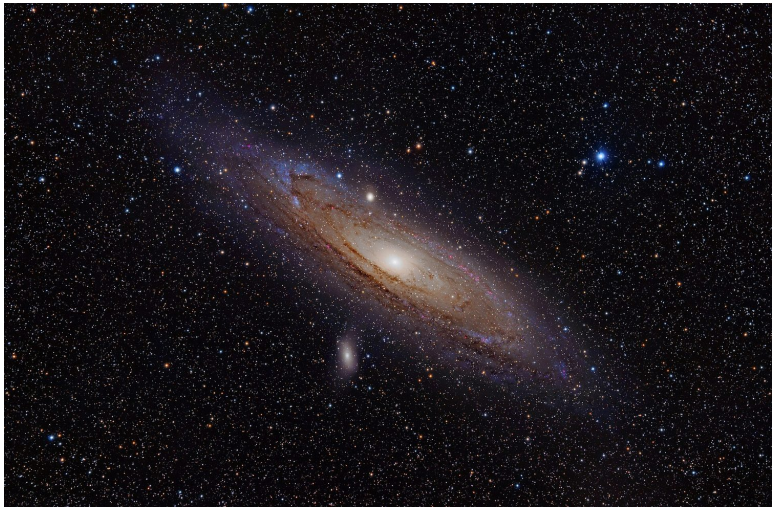
Root first, then power: $\sqrt[4]{81} = 3$ is easy, while 81^3 is not. A negative power never makes the answer negative.



Moving the point 5 places right gives 7.2, so $0.000072 = 7.2 \times 10^{-5}$

Standard form, calculators and sets

Standard form $A \times 10^n$ with $1 \le A < 10$ covers very large and very small measurements; the calculator's $\times 10^x$ key keeps arithmetic exact. Use the calculator "effectively and efficiently": know the trigonometric and square-root functions, keep full precision through the working, and round only at the end.



A galaxy: distances this large are written in standard form, A × 10^n

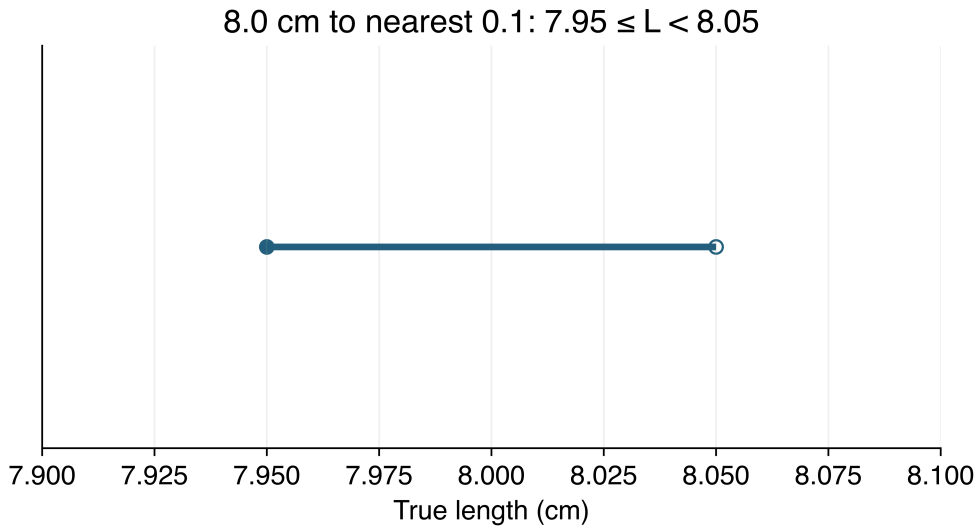
> **Worked example: standard form without a calculator.** Work out $(6\times10^{5})\times(4\times10^{-2})$. Multiply the numbers and add the powers: 24×10^{3}. That is not standard form, because $24\geq10$, so write $24=2.4\times10^{1}$ and the answer is 2.4×10^{4}.

Sets 集合: the language and notation – $n(A)$ for the count, A' the **complement** 补集, $A\cup B$ the **union** 并集, $A\cap B$ the **intersection** 交集 – with **Venn diagrams** 韦恩图 to organise counts (the same toolkit as the probability unit).

Rounding and bounds

Round to an appropriate degree of accuracy – **decimal places** 小数位 and **significant figures** 有效数字 – and know each rounded value carries an interval: to the nearest unit u, the value lies within $\frac{u}{2}$ either side. The Extension addition CALCULATES with upper and lower bounds – the **upper bound** 上界 and **lower bound** 下界: choose the extreme the question needs.

> **Worked example: 2E specimen family, adapted.** A floor of 96 m^2 is measured to 2 s.f.; a safe occupancy needs 5.1 m^2 per person to 1 d.p. The greatest number of people that can be GUARANTEED room: divide the smallest floor by the largest space – LB = 95.5, UB per person = 5.15, so $\frac{95.5}{5.15}=18.5\ldots$; round DOWN to **18**, because 19 people could need $19\times5.15=97.85$ m^2, more than the floor may be. (The specimen's own numbers give 19.8…, also rounded down, to 19.) State which bound each number needs BEFORE dividing.



8.0 cm to the nearest 0.1 cm: 7.95 ≤ L < 8.05

Fractions, decimals and percentages as operators

Equivalent fractions 等值分数 cancel; fractions, decimals and percentages all act as OPERATORS: 35% of 240 is $0.35\times240=84$; $\frac{3}{5}$ of a quantity divides it by 5 and multiplies by 3. Express one quantity as a fraction or percentage of another: 7 as a percentage of 28 is $\frac{7}{28}\times100=25\%$. The Extension addition converts between fractions and **recurring decimals** 循环小数 in both directions – $0.\overline{45}=\frac{45}{99}=\frac{5}{11}$ by the algebraic shift-and-subtract method (specification line, not sampled by the specimens).

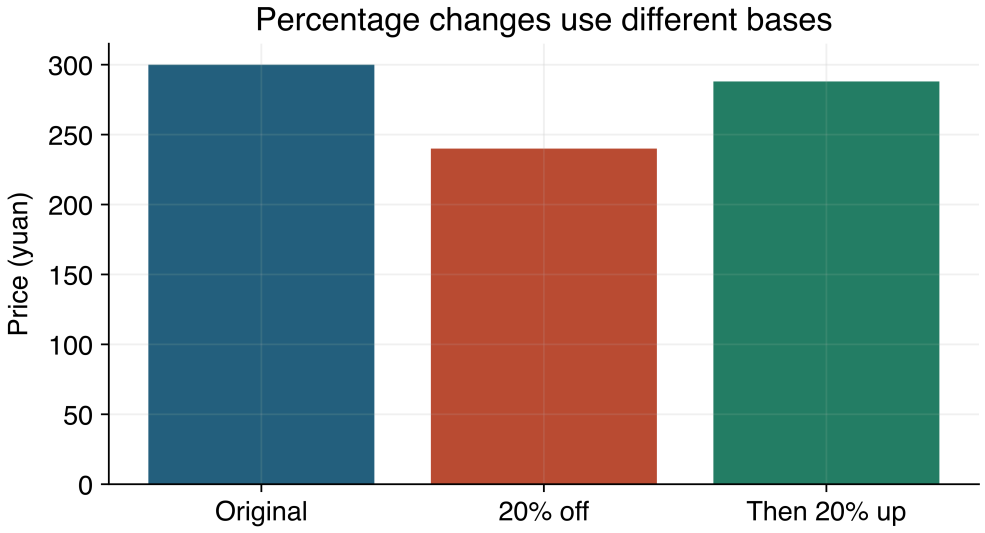
> **Worked example: a recurring decimal with a delay.** Write $0.2\overline{7}$ (that is $0.2777\ldots$) as a fraction in its simplest form. Let $x=0.2\overline{7}$. Move the point until the repeating part starts, then one more cycle:
>
> $$10x=2.777\ldots,\qquad 100x=27.777\ldots\ \Rightarrow\ 90x=25\ \Rightarrow\ x=\frac{25}{90}=\frac{5}{18}.$$
>
> Subtract the two lines whose decimal parts match exactly ($100x$ and $10x$), not x itself, whose decimal part starts with the 2.

Percentage change and compound interest

Percentage change uses **multipliers** 乘数: increase by 12% is ×1.12; decrease by 17% is ×0.83. The Extension additions cover **reverse percentage** 反向百分比 (percentages) problems and the **compound interest formula** 复利公式 $A=P\left(1+\frac{r}{100}\right)^{n}$: £2000 at 3% for 4 years gives $2000\times1.03^{4}=$ £2251.02; reverse: a £92 price after a

15% increase means an original of $92 \div 1.15 = £80$ (specification lines, not sampled by the specimens).

> **Worked example: reverse percentage.** In a sale, a coat costs £68 after a 15% reduction. Find the original price. The sale price is 85% of the original, so original $\times 0.85 = 68$ and the original is $68 \div 0.85 = £80$. The usual mistake is to add 15% of £68 ($68 \times 1.15 = £78.20$): the 15% was of the ORIGINAL price, not the sale price.

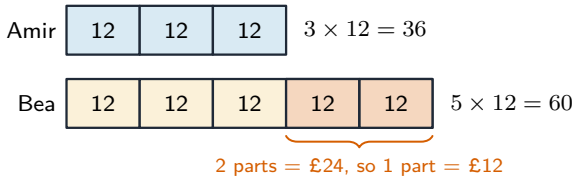


20% off then 20% up gives 288, not 300: the second 20% is of 240

Ratio, rate and proportion

Ratio notation simplifies by dividing through and links to fractions (share £350 in 2 : 5: parts of $\frac{2}{7}$ and $\frac{5}{7}$ – £100 and £250); apply ratio to problems (recipes, maps, mixtures) and use common **rates** 比率 including rates of pay (£9.20 per hour for 7.5 hours = £69) and best-buy comparisons (unit pricing – specification lines, not sampled). **Direct proportion** 正比例 scales both quantities ($y = kx$); **inverse proportion** 反比例 scales them oppositely ($y = \frac{k}{x}$).

> **Worked example: a ratio from a difference.** Amir and Bea share money in the ratio 3 : 5. Bea gets £24 more than Amir. The difference is $5 - 3 = 2$ parts, so 2 parts are £24 and 1 part is £12. Amir gets $3 \times 12 = £36$ and Bea gets $5 \times 12 = £60$. Check: $60 - 36 = 24$. ✓



The difference is 2 parts: 2 parts = 24 pounds, so each part is 12 pounds

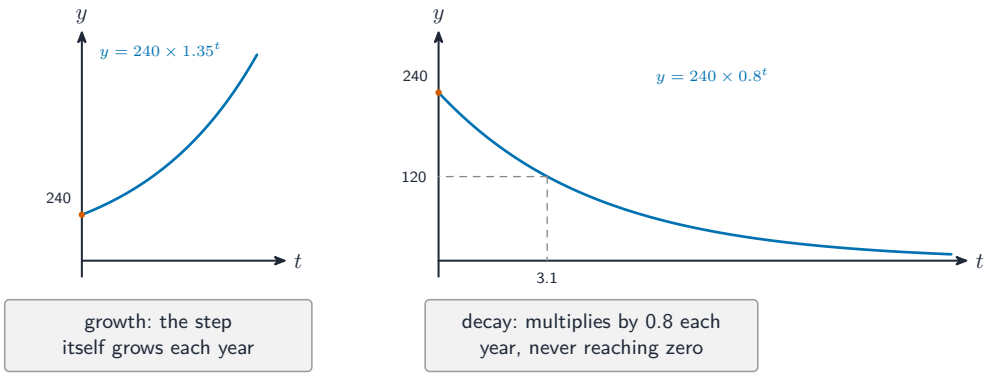
> **Worked example: inverse square proportion.** y is inversely proportional to x^2, and $y = 5$ when $x = 2$. Find y when $x = 4$. Write $y = \frac{k}{x^2}$; then $5 = \frac{k}{4}$, so $k = 20$. When $x = 4$, $y = \frac{20}{16} = 1.25$. Doubling x divides y by $2^2 = 4$, a quick check.

Exponential growth and decay

The final Extension addition models **exponential** 指数 growth and decay: $y = ab^t$ with $b > 1$ growing and $0 < b < 1$ decaying. The 2E specimen fits a decay model from data – successive values 240, 192, 153.6 have ratio 0.8, so $y = 240 \times 0.8^t$ (the specimen's fitted a-family) – and SOLVES with it: £12000 losing 20% per year, $V = 12000 \times 0.8^t$, first falls below £2500 when $0.8^t < 0.2083$, i.e. $t > \ln 0.2083 / \ln 0.8 \approx 7.0$: during the 8th year (the specimen's "how many years" answer family). Growth works identically: £1200 at 7% passes £2000 after $\ln(5/3)/\ln 1.07 \approx 7.6$, i.e. 8 full years.



A growing population: each year multiplies the last by the same factor



Left: exponential growth, $y = 240$ times 1.35 to the power t, rising ever faster. Right: exponential decay, $y = 240$ times 0.8 to the power t, falling towards zero without reaching it.

Checklist

- Order integers, decimals and fractions with $=$, $\neq$, $<$, $>$ on the number line (N1).
- Four operations with formal methods on integers, decimals, fractions, mixed numbers; inverse-operation cancellation (N2, N3).
- Even/odd/prime/factors/multiples vocabulary; HCF and LCM from index-form prime factorisations (N4, Extension; 1E specimen HCF 84).
- Integer powers and roots; index laws; fractional powers; exact surd arithmetic incl. rationalising (N5–N7, Extension).
- Standard form; effective calculator use (N8, N10).
- Sets: $n(A)$, A', $\cup$, $\cap$ with Venn diagrams (N9).
- Rounding to appropriate accuracy; upper/lower bound calculations choosing the right extremes (N11, Extension; 2E specimen 19).
- Equivalent fractions; FDP as operators; fraction/percentage of; recurring decimal conversions (N12–N14, Extension).
- Percentage change with multipliers; reverse percentage; compound interest formula (N15, Extension).
- Ratio notation and sharing; ratio problems; rates of pay and unit pricing (N16–N19).
- Direct and inverse proportion; exponential growth and decay models fitted and solved (N20, Extension; 2E specimen family).

Boundary notes: NO formulae sheet is provided in 9260 – the recall list above is examinable as memory. Lines marked “not sampled” (fractional powers, recurring decimals, reverse percentage, rates of pay and best buys) are taught from the specification text as originals; the specimen pair is Extension-tier only, so Core-tier demand (papers 1–2 Core, 1 h 30, 80 marks) has NO local evidence and holds to spec-text teaching only. Worked examples are adapted from the 2016 specimen pair or original.

Algebra

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OxfordAQA 9260 unit 3.2 (specification for first assessment 2018) – the largest unit at 35% of each Extension paper. Core rows A1–A26 cover notation through graphs, equations, inequalities and sequences; the **Extension additions** add subject-twice rearrangement, triple expansions, all quadratic factorisation, fractional powers, quadratic algebraic fractions, formal proof, the full functions toolkit (domain, range, **inverse** 反函数, **composite** 复合函数), line equations, exponential and trigonometric graphs, **differentiation** 微分 with tangents and **stationary points** 驻点, completing the square, gradients and areas under graphs, variation, every simultaneity pair, quadratic inequalities and quadratic sequences. The 2E specimen samples this unit heavily (algebra is its spine); the unsampled lines are labelled specification originals.

Must-know for unit 3.2 (NO formulae sheet):

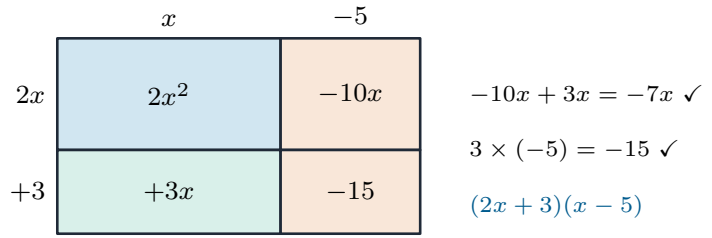
Family	Formulae
quadratic	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $a(x+b)^2 + c$ gives the turning point
line	$y - y_1 = m(x - x_1)$; gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
calculus	$\dfrac{d}{dx}(kx^n) = knx^{n-1}$; tangent: the gradient at the point
nth terms	arithmetic $an + b$; quadratic $an^2 + bn + c$

Notation, substitution and formulae

Letters express generalised number; substitute numbers into formulae and expressions (positive, negative, fractional). The Extension addition rearranges **complex formulae including when the subject appears twice**: from $ax + 3 = 5(xt) - 2a$, collect the subject's terms first – $ax + 2a = 5xt - 3$, so $a(x + 2) = 5xt - 3$ and $a = \dfrac{5xt - 3}{x + 2}$. Check every rearrangement numerically.

Expanding, factorising and algebraic fractions

Collect like terms; expand single brackets and (Extension) products of two or three **binomials** 二项式: $(x+2)(x-3)(x+1) = (x^2-x-6)(x+1) = x^3-7x-6$. Factorise by common factors and (Extension) every quadratic $ax^2 + bx + c$, including the difference of two squares: $2x^2 - 7x - 15 = (2x + 3)(x - 5)$ and $9x^2 - 25 = (3x - 5)(3x + 5)$. Index laws run to fractional powers: $4c^5 \div 3d^3$ stays as $\frac{4c^5}{3d^3}$ in index-form answers (the 1E specimen's answer shape).



Check a factorisation with a grid: the two x-terms must add to -7x and the corner must be -15

Rational expressions 有理式 (Extension) add, subtract, multiply and divide linear or quadratic algebraic fractions – factor first, cancel whole factors, common denominators. The 2E specimen's $\dfrac{11m - 14}{m^2 - 3m - 4}$ and $\dfrac{2x + 3}{x + 4}$ answers come from this machinery.

> **Worked example: simplify, then subtract.** (a) Simplify $\dfrac{x^2 - 9}{x^2 + x - 6}$. (b) Write $\dfrac{3}{x + 2} - \dfrac{1}{x - 1}$ as a single fraction.

(a) Factorise top and bottom, then cancel the whole common factor $(x + 3)$:

$$\frac{x^2 - 9}{x^2 + x - 6} = \frac{(x - 3)(x + 3)}{(x + 3)(x - 2)} = \frac{x - 3}{x - 2}.$$

(b) The common denominator is $(x+2)(x-1)$. Put the second numerator in a bracket so the minus sign reaches both terms:

$$\frac{3(x - 1) - (x + 2)}{(x + 2)(x - 1)} = \frac{3x - 3 - x - 2}{(x + 2)(x - 1)} = \frac{2x - 5}{(x + 2)(x - 1)}.$$

Cancelling single terms (such as the x^2 in part (a)) is never allowed: only whole factors cancel.

Algebraic proof

Argue mathematically that **identities** 恒等式 hold for every value – with the Extension demand for formal **proofs** 证明. Structure: state the algebra, simplify, compare with the claim. Classic (2E specimen family): prove that $n^2 + n + 9$ is odd for every integer n. Write $n^2 + n = n(n + 1)$: a product of two **consecutive integers** 连续整数, one of which is even, so $n(n + 1)$ is even, and even $+9$ is odd. Another parity proof: $(2n + 1)^2 - (2n - 1)^2 = (4n^2 + 4n + 1) - (4n^2 - 4n + 1) = 8n$ – a multiple of 8 for every integer n. Numeric examples are never a proof.

Functions

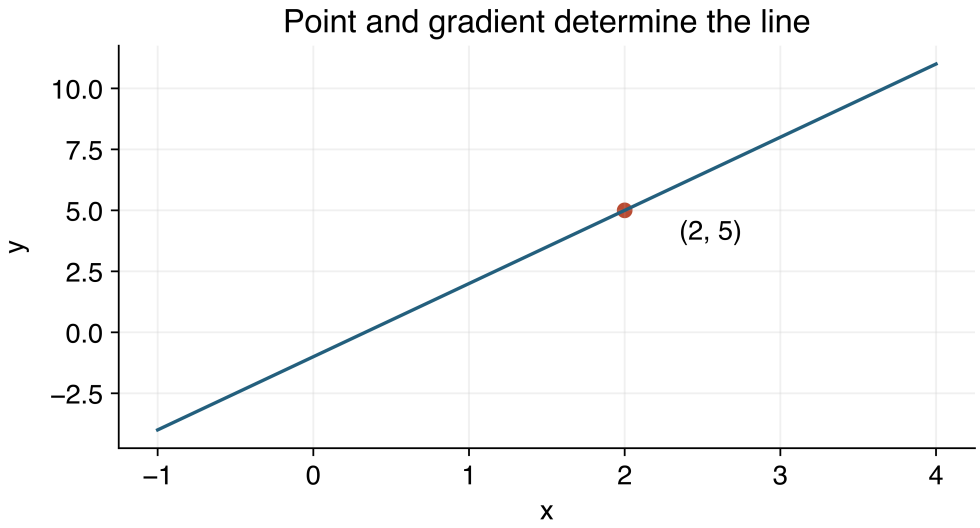
A **function** 函数 maps each input to exactly one output (Core: simple input–output machines; Extension: the formal definition, $\mathrm{f}(x) = \ldots$ notation, **domain** 定义域 and **range** 值域). Composite: $\mathrm{fg}(x) = \mathrm{f}(g(x))$, apply g first – the 2E specimen builds $\mathrm{fg}(x) = (x-5)^2 + 1$ and solves $\mathrm{fg}(x) = 5$: $(x-5)^2 = 4$ gives $x = 3$ (or $x = 7$; check: $(3-5)^2 + 1 = 5$). Inverse: swap and rearrange – the 1E specimen's $\mathrm{f}^{-1}(x) = \frac{x}{3}$ comes from $\mathrm{f}(x) = 3x$.

Straight-line graphs and coordinates

Coordinates in all four **quadrants** 象限; plot $y = mx + c$; the Extension additions find the equation through two points or through one point with a given gradient, and use $y - y_1 = m(x - x_1)$: through $(1, 7)$ and $(2, 9)$, $m = 2$ and $y = 2x + 5$. **Parallel lines** 平行线 share m; **perpendicular** 垂直 gradients multiply to -1 (specification line, not sampled).



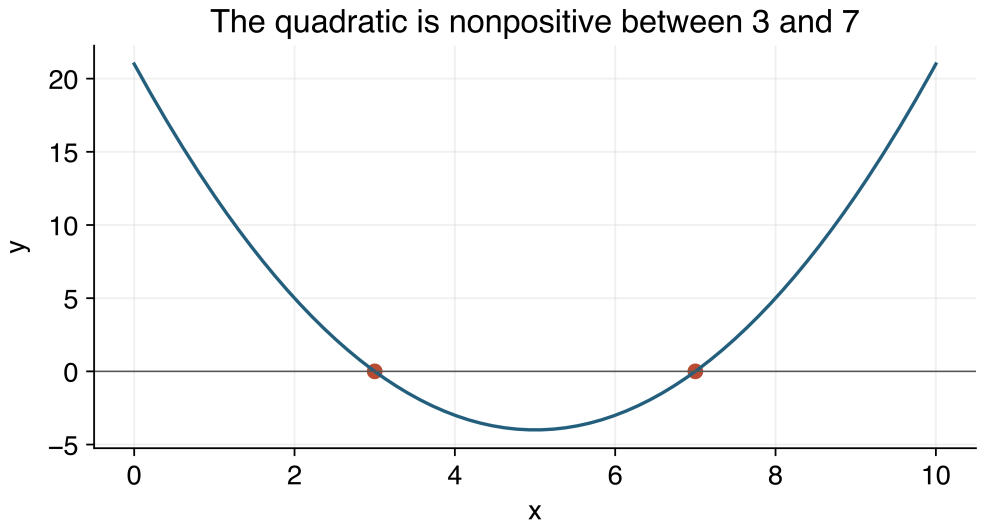
A steep mountain road: gradient is rise over run



Gradient 3 through (2, 5): $y - 5 = 3(x - 2)$, so $y = 3x - 1$

Curves and their graphs

Sketch and interpret linear, quadratic, **cubic** 三次 and reciprocal graphs; the Extension additions add **exponential** functions $y = k^x$ for positive k and the trigonometric graphs (degrees) – both specification lines not sampled by the specimens. Identify roots, **intercepts** 截距 and **turning points** 转折点; the Extension addition DEDUCES turning points by **completing the square** 配方: $y = x^2 - 6x + 5 = (x-3)^2 - 4$ has its minimum at $(3, -4)$ (specification line, not sampled). Gradients and areas under graphs (including quadratic and non-linear graphs) are estimated by tangents and **trapezia** 梯形, and – at Extension – computed exactly by differentiation (next section).



$x^2 - 10x + 21 = (x-3)(x-7) \le 0$ for $3 \le x \le 7$

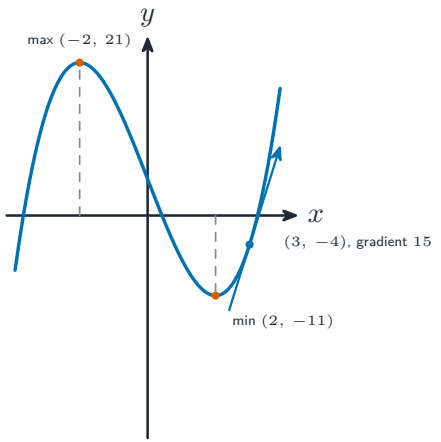
Differentiation

The Extension calculus block: understand and use the **gradient function** 导函数 $\frac{dy}{dx}$, differentiate kx^n for positive integer n (and sums), and know that the gradient of a function at a point IS the gradient of the **tangent** 切线 there – then work out the tangent's equation.

$$y = x^2 \Rightarrow \frac{dy}{dx} = 2x; \quad \text{at } x = 2:\ m = 4, \quad y - 4 = 4(x-2) \Rightarrow y = 4x - 4$$

The 2E specimen's values (family citation): a curve with $\frac{dy}{dx} = 15$ at the point carries tangent $y = 15x - 36$. Stationary points solve $\frac{dy}{dx} = 0$, classified as **maxima, minima** 极值 or **points of inflection** 拐点, with curves sketched from the **derivatives** 导数:

> **Worked example: stationary points and a tangent.** $y = x^3 - 12x + 5$: $\frac{dy}{dx} = 3x^2 - 12 = 0$ gives $x = \pm 2$. At $x = -2$, $y = 21$ (gradient changes + to −: maximum); at $x = 2$, $y = -11$ (changes − to +: minimum). At $x = 3$ the gradient is $3(9) - 12 = 15$ and the tangent is $y + 4 = 15(x-3)$, i.e. $y = 15x - 49$.



tangent (blue) $y = 15x - 49$;
stationary where $3x^2 - 12 = 0$

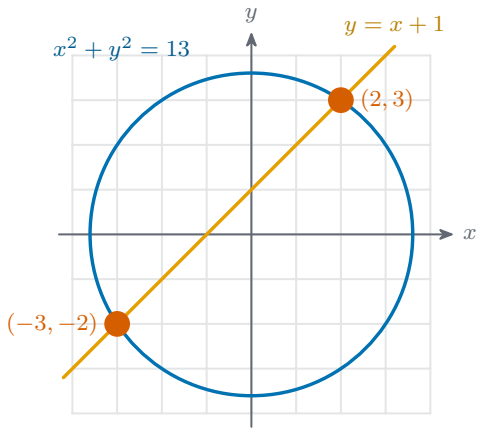
The cubic $y = x$ cubed minus 12x plus 5 with its maximum (-2, 21), minimum (2, -11) and the tangent at (3, -4) with gradient 15 drawn.

Equations and inequalities

Solve linear equations (unknown both sides), quadratics by factorising and (Extension) by completing the square and the quadratic formula – the 1E specimen's $x = -3 \pm \sqrt{7}$ comes from $x^2 + 6x + 2 = 0$: $x = \frac{-6 \pm \sqrt{36-8}}{2}$. Simultaneous pairs: two linear equations, and (Extension) one linear with one **quadratic** 二次方程. Variation (Extension): direct $y = kx^n$ and inverse $y = \frac{k}{x^n}$ in algebraic terms (specification line, not sampled). **Worked example: a linear and a quadratic.** Solve $y = x + 1$ and $x^2 + y^2 = 13$ together. Substitute the linear equation into the quadratic:

$$x^2 + (x+1)^2 = 13 \ \Rightarrow\ 2x^2 + 2x - 12 = 0 \ \Rightarrow\ x^2 + x - 6 = 0 \ \Rightarrow\ (x+3)(x-2) = 0.$$

So $x = 2$, $y = 3$ or $x = -3$, $y = -2$. Give the answers as PAIRS, and find each y from the linear equation: squaring $y^2 = 9$ would wrongly allow $y = -3$ with $x = 2$.

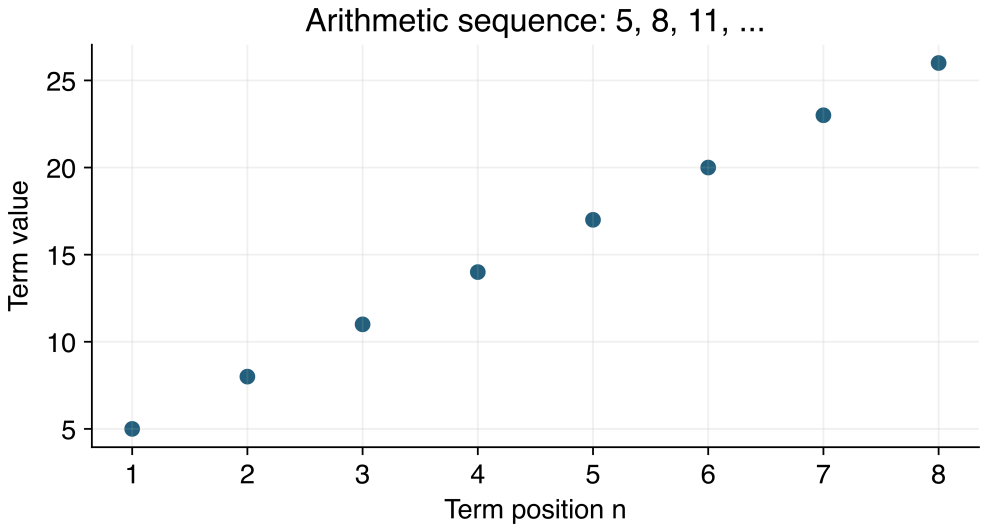


A linear-quadratic pair meets in two points, so the answers come as two (x, y) pairs

Inequalities: linear in one variable, and (Extension) in one or two variables plus **quadratic inequalities** 二次不等式 – solve $x^2 - x - 6 > 0$ by critical values $x = -2, 3$: positive quadratic, so $x < -2$ or $x > 3$ (specification line, not sampled).

Sequences

Generate terms from term-to-term and position-to-position rules; use triangular, square and cube number sequences (Core) with the Extension additions covering **quadratic sequences** 二次数列: **second differences** 二阶差 are constant, and the **nth term** 第 n 项 is $an^2 + bn + c$ with $a = \frac{1}{2}$(second difference). For $5, 9, 17, 29, 45, \ldots$: first differences $4, 8, 12, 16$, second difference 4, so $a = 2$. Take $2n^2$ (which gives $2, 8, 18, 32, 50$) and subtract: the residuals $3, 1, -1, -3, -5$ are linear, $-2n + 5$. So the nth term is $2n^2 - 2n + 5$ – check $n = 4$: $32 - 8 + 5 = 29$. ✓



Add 3 each time: nth term 3n + 2, so the eighth term is 26

Checklist

- Generalised notation; substitution; rearrangement with the subject twice (A1–A3, Extension).
- Expanding two or three binomials; factorising all quadratics and the difference of two squares; fractional index answers; quadratic algebraic fractions (A4–A7, Extension; 1E/2E specimens).
- Formal algebraic proof (A8, Extension; 2E $n^2 + n + 9$ family).
- Functions: definition, domain, range, composite fg, inverse (A9, Extension; 2E $fg(x) = (x-5)^2 + 1$, 1E $f^{-1}(x) = x/3$).
- Coordinates; line equations through two points or point + gradient; perpendicular gradients (A10–A11; the latter not sampled).
- Linear, quadratic, cubic, reciprocal, exponential and trigonometric graphs; turning points by completing the square; gradient/area estimates (A12, A16–A17; several lines not sampled).
- Differentiation of kx^n; gradient = tangent gradient; tangent equations; stationary points classified with sketches (A13–A15, Extension; 2E tangent family).
- Linear equations; quadratics by all three methods; linear–linear and linear–quadratic simultaneity (A19–A21, Extension).
- Context into algebra (A22, Extension); linear (one/two variable) and quadratic inequalities with solution sets (A23, Extension; not sampled).
- Sequence types; nth terms linear and quadratic (A24–A26, Extension).

Boundary notes: NO formulae sheet in 9260. Lines marked "not sampled" (perpendicular gradients, exponential/trig sketches, completing-the-square turning points, variation, quadratic and two-variable inequalities) are specification originals; the specimens are Extension-tier only, so Core-tier demand has no local evidence. Worked examples are adapted from the 2016 specimen pair or original.

Geometry and measures

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OxfordAQA 9260 unit 3.3 (specification Version 5.1) – at 31.5% of the combined specimen marks. Core rows G1–G26 cover angle facts through constructions, mensuration, similarity, trigonometry, transformations, vectors and matrices; the **Extension additions** add the full circle-theorem programme with proof, congruence conditions, bearings, sphere/pyramid/cone/frustum formulae, k-k^2-k^3 similarity, sectors, 3D Pythagoras and trigonometry, the sine/cosine rules with ½ab sinC, combined transformations with fractional/negative scale factors, vector geometry, and the 2×2 matrix programme (identity, unit-square transformations, combinations). The 1E specimen is the geometry paper; unsampled lines are labelled specification originals.

Must-know for unit 3.3 (NO formulae sheet):

Family	Formulae
circle	**circumference** 圆周 $2\pi r$; area πr^2; **arc** 弧 $\frac{\theta}{360}(2\pi r)$; sector $\frac{\theta}{360}\pi r^2$
solids	sphere $V = \frac{4}{3}\pi r^3$, $SA = 4\pi r^2$; cone $V = \frac{1}{3}\pi r^2 h$; pyramid $V = \frac{1}{3}Ah$ (Extension)
triangles	$a^2 = b^2 + c^2 - 2bc\cos A$; $\frac{a}{\sin A} = \frac{b}{\sin B}$; Area $= \frac{1}{2}ab\sin C$

Angles, polygons and symmetry

Conventional terms: points, lines, vertices, edges, planes; parallel, perpendicular; **polygons** 多边形 and **regular polygons** 正多边形; the standard triangle labelling. Angle facts: at a point (360°), on a line (180°), **vertically opposite** 对顶角; parallel lines give **alternate** 内错角, **corresponding** 同位角 and **co-interior** angles (use these names in reasons, never "Z angles"); triangle sum 180° with the **isosceles** 等腰, **equilateral** 等边 and **scalene** 不等边 families. Polygon sums: interior $(n-2)\times 180°$, exterior 360°; for regular polygons also the angle at the centre $= \frac{360°}{n}$. The special quadrilaterals (square, rectangle, parallelogram, trapezium, kite, **rhombus** 菱形) carry their side/angle/diagonal properties, and 2D shapes have reflection and rotation **symmetry** 对称 counts (a regular hexagon: 6 lines, order 6).

A length scale factor is squared for area



Length scale factor 3 gives area scale factor 9: 12 × 9 = 108

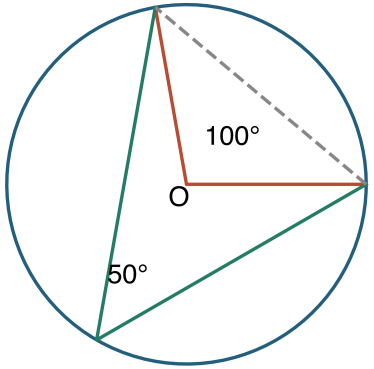
Congruence, similarity and circle theorems

Congruence 全等 with the Extension conditions for triangles – SSS, SAS, ASA, RHS – each justified by which three independent measurements pin a triangle. **Similarity** 相似 scales lengths by k: (Extension) areas by k^2, volumes by k^3, in every direction.



Nesting dolls are similar shapes: the same shape at different sizes

The centre angle is twice the circumference angle



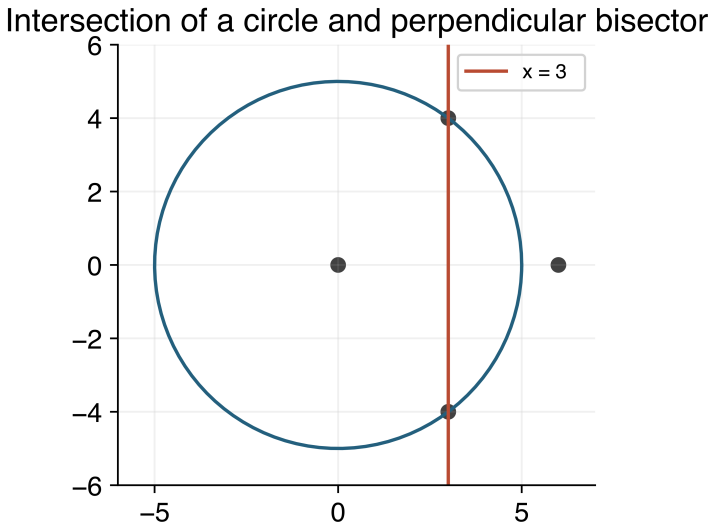
The angle at the centre is twice the angle at the circumference: 100° and 50°

The Extension circle programme: angle at the centre is twice the angle at the circumference; the **semicircle** 半圆 angle is 90°; angles in the same segment are equal; opposite angles of a **cyclic quadrilateral** 圆内接四边形 sum to 180°; tangent ⊥ **radius** 半径; tangents from an external point are equal; the perpendicular from the centre bisects a **chord** 弦; the **alternate segment** 弦切角 theorem. The verified chain: an angle at the circumference of 35° subtends the same arc as a centre angle of 70°, and the angle from the OPPOSITE arc is $\frac{1}{2}(360° - 70°) = 145°$ – the two circumference angles are opposite angles of a cyclic quadrilateral ($35° + 145° = 180°$) – every step names its theorem (the 1E specimen's circle question carries answers of this family; G9's geometric **proof** 证明 demand).

3D shapes, constructions and bearings

Faces, surfaces, edges and vertices of cubes, cuboids, **prisms** 棱柱, **cylinders** 圆柱, pyramids, cones and spheres; **plans and elevations** 平面图和立面图 (front, side, top views – a specification line not sampled by the specimens). Measure line segments and angles; scale drawings with scale factors; **bearings** 方位角: three-figure, clockwise from north (Extension includes the eight compass points) – the 2E specimen's 252° comes from a bearings chain: a return bearing plus an angle at a point.

Worked example: an angle from two bearings. B is on a bearing of 070° from A, and C is on a bearing of 160° from B. Find angle ABC. The bearing of A FROM B is the return bearing $070° + 180° = 250°$. At B, the direction to A is 250° and the direction to C is 160°, so angle $ABC = 250° - 160° = 90°$. Draw a north line at EVERY point you work from; bearings are measured at the point named after "from".



Points 5 from O and equidistant from (0, 0) and (6, 0): $x = 3$ meets the circle at (3, ±4)

Ruler-and-compass **constructions** 作图: **perpendicular bisector** 垂直平分线 of a segment, perpendicular to a line from/at a point, **angle bisector** 角平分线, 60° angle – used to build figures and solve **loci** 轨迹 problems; the perpendicular distance from a point to a line is the shortest. Leave all arcs visible: the arcs are the working.

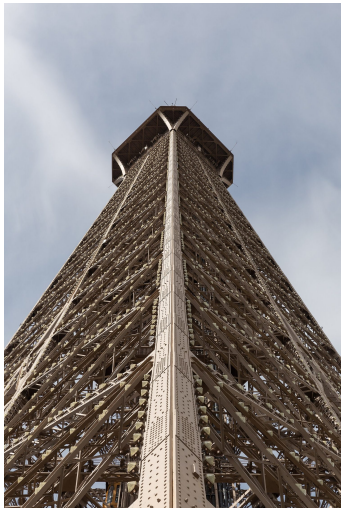
Units, mensuration and similarity ratios

Standard units with conversions (time 12/24-hour; length, area, volume/capacity, mass; **compound units** 复合单位 speed and **density** 密度). Areas: triangle $\frac{1}{2}bh$, parallelogram bh, trapezium $\frac{1}{2}(a+b)h$; volume by $V = Ah$ (constant **cross-section** 横截面). The circle: $2\pi r$, πr^2; (Extension) **sectors** 扇形 arc $\frac{\theta}{360} \times 2\pi r$ and area $\frac{\theta}{360} \times \pi r^2$, plus sphere/pyramid/cone **surface** and **volume** formulae and composite solids including **frustums** 圆台 (specification lines, several not sampled) – a cone of $r = 5$, $h = 12$ has slant 13, $V = 100\pi$ and curved surface 65π; the 1E specimen's cone problem finds a proportion $p = 15$.

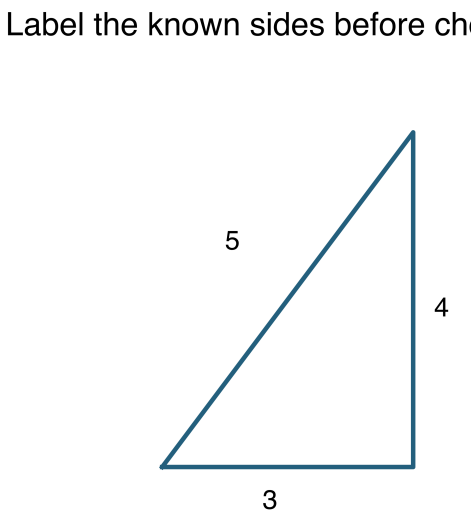
Worked example: a sector. A sector has radius 6 cm and angle 150°. The arc is $\frac{150}{360} \times 2\pi \times 6 = 5\pi = 15.7$ cm, so the perimeter is $5\pi + 6 + 6 = 27.7$ cm (both radii count). The area is $\frac{150}{360} \times \pi \times 6^2 = 15\pi = 47.1$ cm^2. Similar figures (Extension): lengths k : areas k^2 : volumes k^3 – the 2E specimen's similar solids give 864 cm^3 from a volume ratio.

Pythagoras, trigonometry and 3D problems

$a^2 + b^2 = c^2$ with SOH CAH TOA in right-angled triangles – 2D always, 3D figures at Extension: extract the right triangle first (the 1E specimen's 3D trig gives 62.7°). The **sine rule** 正弦定理 $\frac{a}{\sin A} = \frac{b}{\sin B}$ and **cosine rule** 余弦定理 $a^2 = b^2 + c^2 - 2bc\cos A$ cover any triangle (Extension), with Area $= \frac{1}{2}ab\sin C$ reversible – the 2E specimen's cosine rule gives 102.6°, and $\cos x = \frac{8}{11}$ yields $x = 43.3°$.



Looking up at a tower: the angle of elevation and the distance give the height using tan

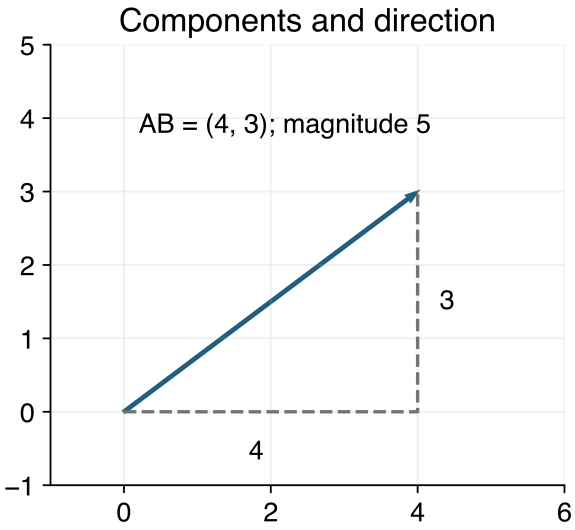


Label opposite, adjacent and hypotenuse first: 3, 4 and 5 here

Worked example: 2E cosine-rule family, adapted. A triangle has $a = 9$, $b = 11$ and angle $C = 72°$. $c^2 = 81 + 121 - 2(9)(11)\cos 72° = 202 - 198 \times 0.309 = 202 - 61.2 = 140.8$, so $c = 11.9$ (3 s.f.). Then the smallest angle sits opposite the shortest side: $\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{121+140.8-81}{2\times 11\times 11.87} = \frac{180.8}{261.1} = 0.693$, so $A = 46.2°$ (keeping c unrounded) – check $A + B + C = 180°$ with $B = 61.8°$. ✓

Transformations with matrices and vectors

Single rotations, reflections, translations (by a vector) and enlargements (positive factor) – with the preserved properties named; the Extension additions cover **combined transformations** and enlargements by **fractional and negative scale factors** 负比例因子 (an inverted, shrunken image on the opposite side of the centre). **Vectors** 向量: notation, sum, difference and scalar multiples (graphically and by components), the commutative and associative laws, and simple 2D geometry – the 2E specimen proves $CB \parallel MN$ via a $\frac{1}{2}$ ratio using vectors.



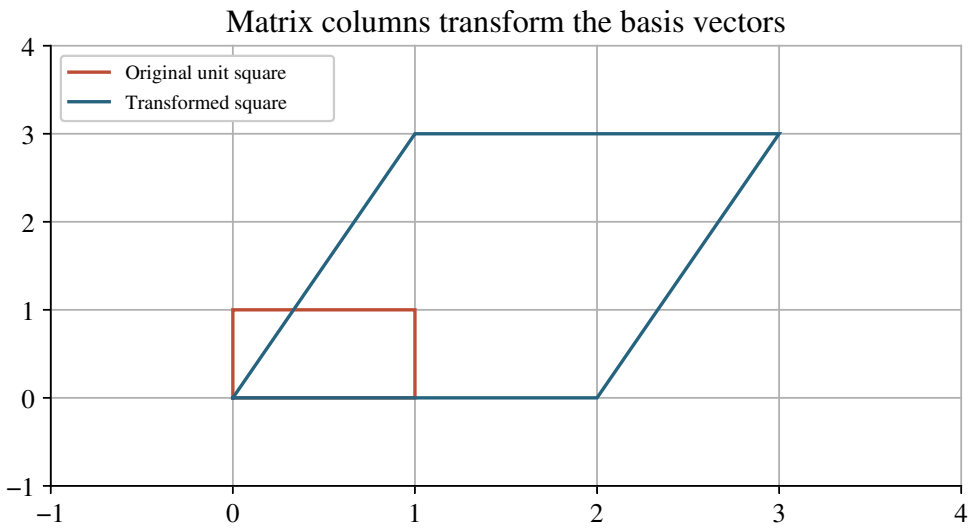
AB = (4, 3) has magnitude $\sqrt{(4^2 + 3^2)} = 5$

The 2×2 **matrix** 矩阵 programme (Extension): multiplication (2×2 by 2×2, by 2×1, scalar), the **identity matrix** 单位矩阵 I; transformations of the UNIT SQUARE – rotations of 90°/180°/270° about the origin, reflections in $x = 0$, $y = 0$, $y = x$, $y = -x$, enlargements centred on the origin – represented by matrices such as $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ (reflection in $y = -x$, the 1E specimen's matrix); combined transformations by matrix multiplication, in the order the transformations apply.

Worked example: a combined transformation. Reflect in $y = x$, matrix $\mathbf{M} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, then rotate 90° anticlockwise about the origin, matrix $\mathbf{R} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. The FIRST transformation sits on the RIGHT:

$$\mathbf{RM} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix},$$

which is a reflection in the y-axis ($x = 0$). Check with a point: $(2, 1)$ reflects to $(1, 2)$, which rotates to $(-2, 1)$, and the reflection in the y-axis also sends $(2, 1)$ to $(-2, 1)$. ✓ Multiplying in the other order gives **MR**, a reflection in the x-axis: the order matters.



The matrix columns (2, 0) and (1, 3) are the images of the basis vectors

Worked example: 1E matrix-solve family, adapted. Solve $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$: the equations are $3x + y = 10$ and $x + 2y = 5$. From the first, $y = 10 - 3x$; substituting: $x + 2(10 - 3x) = 5$, so $-5x = -15$, giving $x = 3$ and $y = 1$. Check: $3(3) + 1 = 10$. ✓ The 1E specimen's own pairs $(6, -\frac{1}{2})$ and $(-2, -\frac{5}{2})$ come from the same method on a second, linked system.

Checklist

- Conventional terms and triangle notation; angle facts; parallel-line angles; polygon interior/exterior/centre sums; quadrilateral properties; symmetry counts (G1–G6).
- Congruence with SSS/SAS/ASA/RHS; similarity with k, k^2, k^3 ratios (G7, G17, Extension).
- Circle language and the full theorem bank with proof-style reasons (G8–G9, Extension; 35° → 70° → 145° verified chain).
- 3D shape properties; plans and elevations (not sampled); measuring, scale factors, bearings (G10–G12).
- Constructions (four ruler-and-compass types) with loci; shortest distance property (G13).
- Units and compound measures; area and V = Ah formulae; circle formulae; sectors; sphere/pyramid/cone/frustum surface and volume; composite shapes (G14–G16, Extension).
- Pythagoras and trigonometry in 2D and 3D; sine/cosine rules and ½ab sinC (G19–G20, Extension; 1E 62.7°, 2E 102.6° and cos x = 8/11).
- Transformations including combined and fractional/negative enlargements (G21, Extension).
- Vector notation and geometry (G22; 2E CB ∥ MN proof).
- Matrix multiplication, identity, unit-square transformations and combinations (G23–G26, Extension; 1E matrices).

Boundary notes: NO formulae sheet in 9260. Lines marked "not sampled" (plans and elevations, parts of the solids programme) are specification originals; the specimens are Extension-tier only. Worked examples are adapted from the 2016 specimen pair or original.

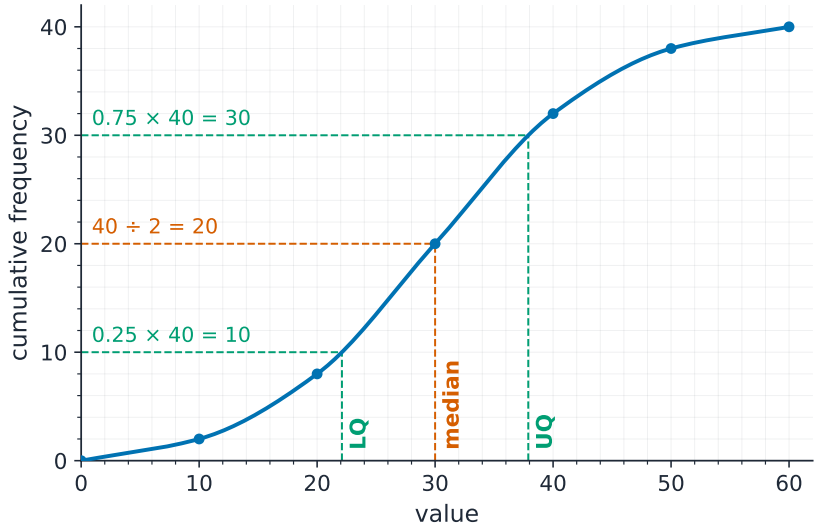
Statistics and probability

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OxfordAQA 9260 unit 3.4 (specification Version 5.1) – the smallest unit at 13% of the combined specimen marks. Core rows S1–S5 cover data types, reading tables, two-way tables, the full chart bank and the basic averages; the **Extension additions** add histograms with UNEQUAL class intervals, cumulative frequency diagrams, box plots, and quartiles with the inter-quartile range and percentiles. The 1E specimen carries a histogram and a cumulative-frequency correction to upper boundaries; the 2E specimen's statistics questions include the 40-men and 38% reads. Lines the specimens never sample are labelled specification originals.

Must-know for unit 3.4:

Family	Formulae
histogram	frequency density $= \dfrac{\text{frequency}}{\text{class width}}$ – bar AREA is the frequency
cumulative frequency	plot at the UPPER class boundary; median at $\frac{n}{2}$, quartiles at $\frac{n}{4}$, $\frac{3n}{4}$
averages	mean $= \dfrac{\sum fx}{\sum f}$ (grouped: midpoints); IQR $= Q_3 - Q_1$



Read the median at n/2 and the quartiles at n/4 and 3n/4

Data types, tables and charts

Qualitative 定性 data categorises (favourite fruit); **discrete** 离散 data counts (number of goals); **continuous** 连续 data measures (height), grouped or ungrouped. Extract data from printed tables and lists – read the row and column headers first, and quote units. **Two-way tables** 双向表 cross-tabulate two variables; every row and column total checks the interior cells: 30 students, 18 cricket, 15 tennis and 7 both fills a 2×2 whose cells must sum to 30.

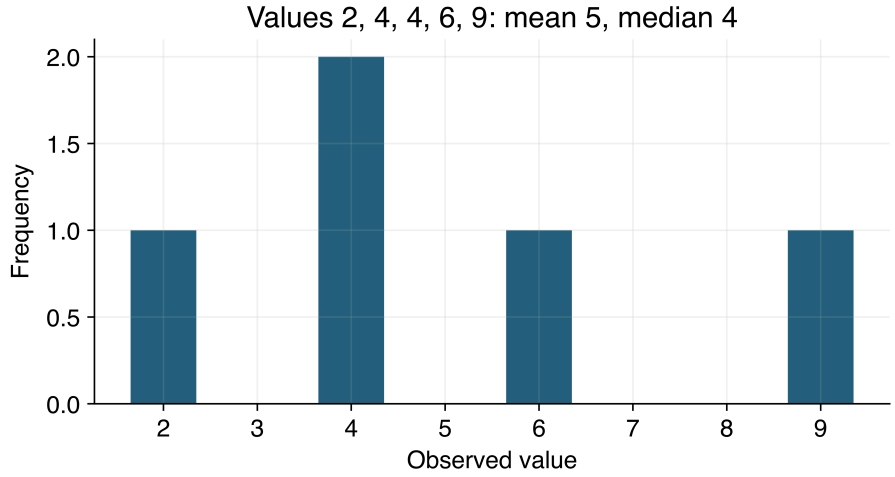
> **Worked example: completing a two-way table.** Start from the cell you know: 7 play both. Then cricket but not tennis is $18 - 7 = 11$, tennis but not cricket is $15 - 7 = 8$, and neither is $30 - 7 - 11 - 8 = 4$.

	Tennis	Not tennis	Total
Cricket	7	11	18
Not cricket	8	4	12
Total	15	15	30

Every row and column adds up, which is the check. Adding $18 + 15 = 33$ and calling it "more than 30, impossible" forgets the 7 counted twice.

The Core chart bank: scatter graphs, stem-and-leaf, tally charts, pictograms, bar charts, dual and composite bar charts, pie charts, line graphs, frequency polygons and histograms with EQUAL class intervals. Choose by data type – categories take bars, comparisons take dual bars, parts-of-a-whole take pie charts, trends take line graphs, correlation takes scatter graphs.

For a pie chart, each item's angle is its fraction of 360°: if 14 of 40 students choose apples, the apple sector is $\frac{14}{40} \times 360° = 126°$. The angles must add to 360°.



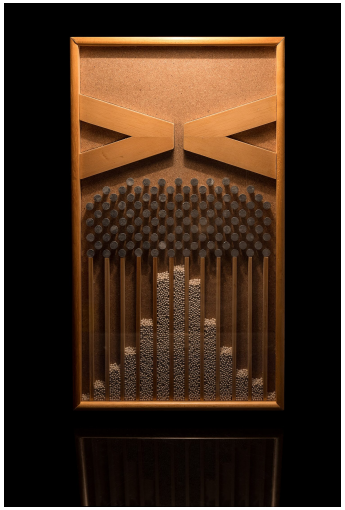
For 2, 4, 4, 6, 9: mean = 25 ÷ 5 = 5, median = 4

Extension diagrams: unequal histograms, CF and box plots

A **histogram** 直方图 with **unequal class intervals** 不等组距 draws bar HEIGHT as the **frequency density** 频率密度 = frequency over class width – the AREA is the frequency (classes of widths 2, 3, 5, 10, 15 holding 5, 12, 18, 14, 9 give heights 2.5, 4, 3.6, 1.4, 0.6). A **cumulative frequency diagram** 累积频率图 plots each running total at its UPPER class boundary as a smooth increasing curve (the 1E specimen's correction is exactly this upper-boundary point placement); the median reads at $\frac{n}{2}$, quartiles at $\frac{n}{4}$ and $\frac{3n}{4}$. A **box plot** 箱线图 (five-number summary: minimum, Q_1, median, Q_3, maximum) shows spread at a glance – the box spans the IQR, the whiskers the range, outliers by judgement.

Averages, quartiles and percentiles

The **mean** 平均数 divides the total by the count (grouped data multiplies each MID-POINT by its frequency first); the **median** 中位数 orders and takes the middle; the **mode** 众数 is most frequent and the **modal class** 众数组 is the class with the highest frequency DENSITY when widths differ. The Extension additions add **quartiles** 四分位数 and the **inter-quartile range** 四分位距 $= Q_3 - Q_1$ (the median of the lower and upper halves), and **percentiles** 百分位数 read like the median at any percentage – the pth percentile is the value $\frac{p}{100}n$ of the way up the ordered data or the cumulative-frequency curve.



A Galton board: the balls pile up around a centre, and the spread shows how far they scatter

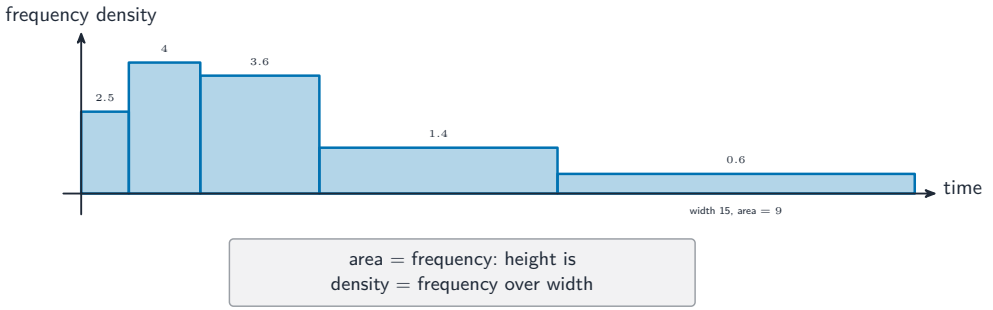
Worked example: an estimated mean. Masses m (kg) of 20 parcels:

Mass	$0 < m \leq 4$	$4 < m \leq 8$	$8 < m \leq 12$	$12 < m \leq 20$
Frequency	5	8	4	3

The midpoints are 2, 6, 10 and 16, so $\sum fx = 10 + 48 + 40 + 48 = 146$ and the estimated mean is $146 \div 20 = 7.3$ kg. It is an estimate because the exact masses are not known. The last class is wider (width 8), so check the modal class by frequency density: $\frac{5}{4} = 1.25$, $\frac{8}{4} = 2$, $\frac{4}{4} = 1$, $\frac{3}{8} = 0.375$; the modal class is $4 < m \leq 8$.

Worked example: reading a cumulative frequency curve. 60 workers' weekly hours have cumulative frequencies 6, 17, 27, 42, 60 at upper boundaries 10, 15, 20, 25, 30 (hours). The median reads at worker 30: between 20 (27) and 25 (42), so $20 + 5 \times \frac{30-27}{42-27} = 21$ hours. Q_1 at worker 15: $10 + 5 \times \frac{15-6}{17-6} = 14.1$; Q_3 at worker 45: $25 + 5 \times \frac{45-42}{60-42} = 25.8$; IQR ≈ 11.7 hours. Draw the reading lines – the marks are for the reads.

Comparing two distributions. Always make TWO comparisons, each in context: one of an average (median) and one of spread (IQR). For example, "Group A's median time (21 h) is higher than Group B's (17 h), so A typically works longer; A's IQR (11.7 h) is larger than B's (6 h), so A's times are less consistent." Comparing only the ranges is weak, because one extreme value can change a range.



A histogram with unequal class widths 2, 3, 5, 10, 15, frequencies 5, 12, 18, 14, 9 and heights marked as the frequency densities 2.5, 4, 3.6, 1.4, 0.6, with the area-is-frequency takeaway.

Checklist

- Data types qualitative/discrete/continuous; grouped and ungrouped (S1).
- Extract from printed tables and lists, quoting units (S2).
- Design and use two-way tables with row/column checks (S3).
- The Core chart bank, chosen by data type (S4).

- Unequal-interval histograms by frequency density (area is frequency); cumulative frequency diagrams at UPPER boundaries with median/quartile reads; box plots (S4 Extension; 1E histogram and CF-boundary anchors).
- Mean (grouped via midpoints), median, mode and modal class (S5).
- Quartiles, inter-quartile range and percentiles from data or the curve (S5 Extension).

Boundary notes: probability vocabulary sits in this specification's Number/Algebra work and earlier units' sets; the AO split is asserted by the specification. Lines not sampled by the specimens (stem-and-leaf, pie charts and several chart types on the Extension papers) are taught as specification originals; the specimens are Extension-tier only, so Core-tier demand has no local evidence. Worked examples are adapted from the 2016 specimen pair or original.