

Course Companion

# OxfordAQA • International GCSE • Mathematics

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Every topic handout in one volume

exercises.lol

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# Number

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OxfordAQA International GCSE Mathematics (9260) unit 3.1 (specification for first assessment 2018). Core rows N1–N20 run from ordering and the four operations through factors and indices, standard form, sets, rounding, FDP operators, percentage change and ratio; the **Extension additions** add index-form factorisation, fractional powers, exact surd work, bounds calculations, recurring decimals, reverse percentages, the compound interest formula and exponential growth and decay. The 2016 specimen pair (1E/2E, Extension tier) samples this unit at HCF, surds, fraction chains, bounds and exponential models; lines the specimens never sample are taught here as specification originals and labelled so.

**Must-know for unit 3.1** (NO formulae sheet in 9260 – the recall list is the specification itself):

| Family             | Formulae                                                                                               |
|--------------------|--------------------------------------------------------------------------------------------------------|
| compound interest  | $A = P \left(1 + \frac{r}{100}\right)^n$ (Extension)                                                   |
| bounds             | value to the nearest unit $u$ : $LB = v - \frac{u}{2}$ , $UB = v + \frac{u}{2}$                        |
| recurring decimals | $x = 0.\overline{45} \Rightarrow 100x = 45.\overline{45} \Rightarrow x = \frac{45}{99} = \frac{5}{11}$ |
| exponential models | growth $y = a b^t$ with $b > 1$ ; decay $0 < b < 1$ (Extension)                                        |

## Ordering and the four operations

Order positive and negative integers, decimals and fractions on the number line with  $=$ ,  $\neq$ ,  $<$ ,  $>$  – convert to one form first when comparing ( $\frac{3}{5} = 0.6$ ). The four operations with formal written methods cover integers, decimals, simple fractions proper and improper, and mixed numbers:  $2\frac{1}{3} + 1\frac{1}{4} = \frac{28}{12} + \frac{15}{12} = \frac{43}{12} = 3\frac{7}{12}$ . Inverse operations cancel and simplify: multiplication undoes division, subtraction undoes addition – use cancellation early ( $\frac{15}{28} \times \frac{7}{9} = \frac{15}{4} \times \frac{1}{9} = \frac{5}{12}$  after cancelling the 7 and 3).

$$72 = 2^3 \times 3^2$$

$$90 = 2 \times 3^2 \times 5$$

Shared smallest powers:  $2 \times 3^2 = 18$

*Original native-lesson illustration; labels belong to its worked example.*

## Factors, primes, HCF and LCM

Even, odd, prime, factor, multiple, common factor, common multiple – with the **highest common factor** and **lowest common multiple** found from prime factorisations. The Extension addition writes every factor in index form:  $504 = 2^3 \times 3^2 \times 7$  and  $588 = 2^2 \times 3 \times 7^2$ , so the HCF is  $2^2 \times 3 \times 7 = 84$  (the 1E specimen's answer) and the LCM is  $2^3 \times 3^2 \times 7^2 = 3528$ .

## Indices, roots and surds

Positive integer powers with square and cube roots ( $\sqrt{144} = 12$ ,  $\sqrt[3]{125} = 5$ ); the index laws multiply and divide by adding and subtracting indices. The Extension addition brings **fractional powers** ( $27^{1/3} = 3$ ,  $8^{2/3} = 4$ : cube-root then square) and **exact calculation with surds** – simplify by extracting squares and rationalise denominators with conjugates:

$$\sqrt{50} - \sqrt{18} + \sqrt{8} = 5\sqrt{2} - 3\sqrt{2} + 2\sqrt{2} = 4\sqrt{2}, \quad \frac{5}{3 - \sqrt{2}} = \frac{5(3 + \sqrt{2})}{9 - 2} = \frac{5(3 + \sqrt{2})}{7}$$

The 1E specimen's surd answer  $4 + 4\sqrt{2}$  comes from this same machinery (family-level citation).

$$0.000072 = 7.2 \times 10^{-5}$$

7.2 is the coefficient; -5 is the power

The coefficient is between 1 and 10

*Original native-lesson illustration; labels belong to its worked example.*

## Standard form, calculators and sets

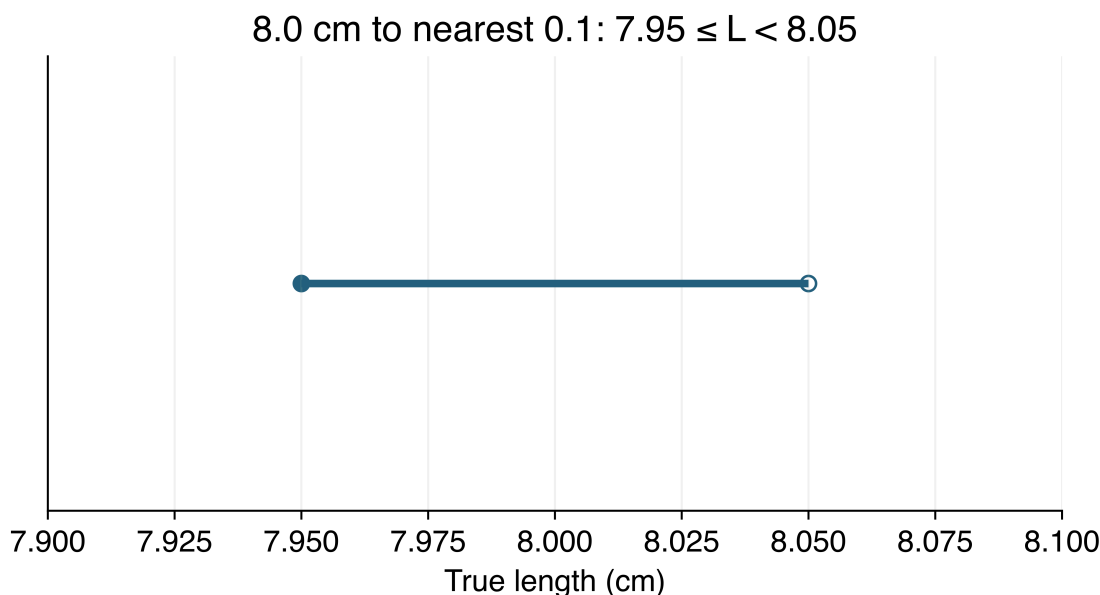
Standard form  $A \times 10^n$  with  $1 \leq A < 10$  covers very large and very small measurements; the calculator's  $\times 10^x$  key keeps arithmetic exact. Use the calculator “effectively and efficiently”: know the trigonometric and square-root functions, keep full precision through the working, and round only at the end.

**Sets** : the language and notation –  $n(A)$  for the count,  $A'$  the complement,  $A \cup B$  the union,  $A \cap B$  the intersection – with Venn diagrams to organise counts (the same toolkit as the probability unit).

## Rounding and bounds

Round to an appropriate degree of accuracy – decimal places and significant figures – and know each rounded value carries an interval: to the nearest unit  $u$ , the value lies within  $\frac{u}{2}$  either side. The Extension addition CALCULATES with upper and lower bounds – the **upper bound** and **lower bound** : choose the extreme the question needs.

**Worked example (2E specimen family, adapted).** A floor of  $96 \text{ m}^2$  is measured to 2 s.f.; a safe occupancy needs  $5.1 \text{ m}^2$  per person to 1 d.p. The LEAST number of people that must fit: divide the smallest floor by the largest space –  $\text{LB} = 95.5$ ,  $\text{UB per person} = 5.15$ , so  $\frac{95.5}{5.15} = 18.5 \dots$ ; you cannot have half a person and everyone must fit, so at least **19** people (the specimen's answer). State which bound each number needs BEFORE dividing.



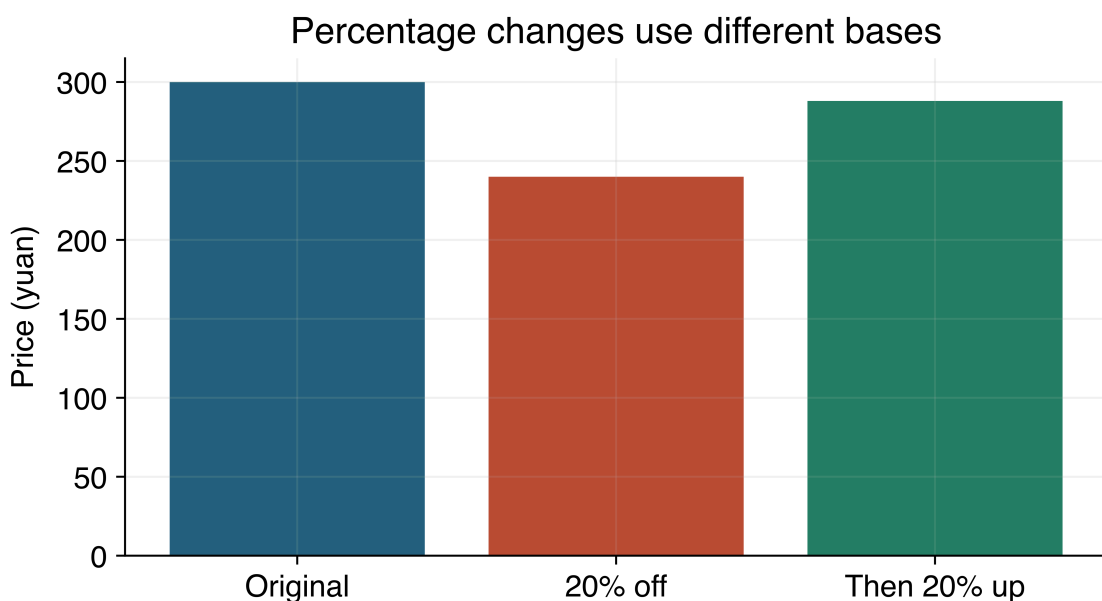
*Original native-lesson illustration; labels belong to its worked example.*

## Fractions, decimals and percentages as operators

Equivalent fractions cancel; fractions, decimals and percentages all act as OPERATORS: 35% of 240 is  $0.35 \times 240 = 84$ ;  $\frac{3}{5}$  of a quantity multiplies by  $\frac{3}{5}$  and divides by 5. Express one quantity as a fraction or percentage of another: 7 as a percentage of 28 is  $\frac{7}{28} \times 100 = 25\%$ . The Extension addition converts between fractions and **recurring decimals** in both directions –  $0.\overline{45} = \frac{45}{99} = \frac{5}{11}$  by the algebraic shift-and-subtract method (specification line, not sampled by the specimens).

## Percentage change and compound interest

Percentage change uses multipliers: increase by 12% is  $\times 1.12$ ; decrease by 17% is  $\times 0.83$ . The Extension additions cover **reverse percentage** (percentages) problems and the **compound interest formula**  $A = P \left(1 + \frac{r}{100}\right)^n$ : £2000 at 3% for 4 years gives  $2000 \times 1.03^4 = \text{£}2251.02$ ; reverse: a £92 price after a 15% increase means an original of  $92 \div 1.15 = \text{£}80$  (specification lines, not sampled by the specimens).



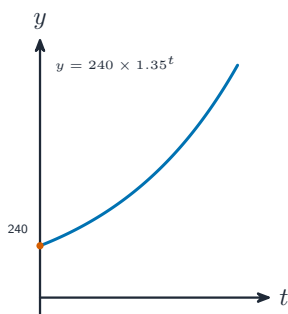
*Original native-lesson illustration; labels belong to its worked example.*

## Ratio, rate and proportion

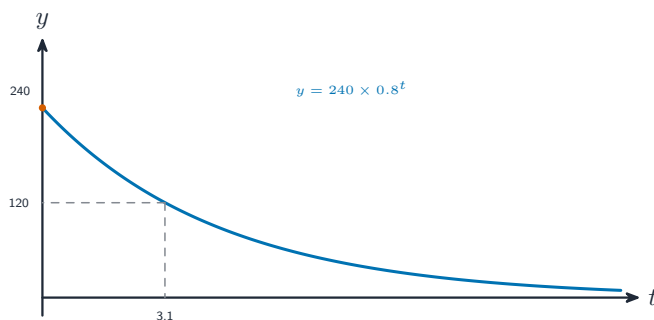
Ratio notation simplifies by dividing through and links to fractions (share £350 in 2 : 5: parts of  $\frac{2}{7}$  and  $\frac{5}{7}$  – £100 and £250); apply ratio to problems (recipes, maps, mixtures) and use common **rates** including rates of pay (£9.20 per hour for 7.5 hours = £69) and best-buy comparisons (unit pricing – specification lines, not sampled). Direct proportion scales both quantities ( $y = kx$ ); inverse proportion scales them oppositely ( $y = \frac{k}{x}$ ).

## Exponential growth and decay

The final Extension addition models **exponential** growth and decay:  $y = ab^t$  with  $b > 1$  growing and  $0 < b < 1$  decaying. The 2E specimen fits a decay model from data – successive values 240, 192, 153.6 have ratio 0.8, so  $y = 240 \times 0.8^t$  (the specimen's fitted  $a$ -family) – and SOLVES with it: £12000 losing 20% per year,  $V = 12000 \times 0.8^t$ , first falls below £2500 when  $0.8^t < 0.2083$ , i.e.  $t > \ln 0.2083 / \ln 0.8 \approx 7.0$ : during the 8th year (the specimen's "how many years" answer family). Growth works identically: £1200 at 7% passes £2000 after  $\ln(5/3) / \ln 1.07 \approx 7.6$ , i.e. 8 full years.



growth: the step  
itself grows each year



decay: multiplies by 0.8 each  
year, never reaching zero

## Checklist

- Order integers, decimals and fractions with  $=$ ,  $\neq$ ,  $<$ ,  $>$  on the number line (N1).
- Four operations with formal methods on integers, decimals, fractions, mixed numbers; inverse-operation cancellation (N2, N3).
- Even/odd/prime/factors/multiples vocabulary; HCF and LCM from index-form prime factorisations (N4, Extension; 1E specimen HCF 84).
- Integer powers and roots; index laws; fractional powers; exact surd arithmetic incl. rationalising (N5–N7, Extension).
- Standard form; effective calculator use (N8, N10).
- Sets:  $n(A)$ ,  $A'$ ,  $\cup$ ,  $\cap$  with Venn diagrams (N9).
- Rounding to appropriate accuracy; upper/lower bound calculations choosing the right extremes (N11, Extension; 2E specimen 19).
- Equivalent fractions; FDP as operators; fraction/percentage of; recurring decimal conversions (N12–N14, Extension).
- Percentage change with multipliers; reverse percentage; compound interest formula (N15, Extension).
- Ratio notation and sharing; ratio problems; rates of pay and unit pricing (N16–N19).
- Direct and inverse proportion; exponential growth and decay models fitted and solved (N20, Extension; 2E specimen family).

Boundary notes: NO formulae sheet is provided in 9260 – the recall list above is examinable as memory. Lines marked “not sampled” (fractional powers, recurring decimals, reverse percentage, rates of pay and best buys) are taught from the specification text as originals; the specimen pair is Extension-tier only, so Core-tier demand (papers 1–2 Core, 1 h 30, 80 marks) has NO local evidence and holds to spec-text teaching only. Worked examples are adapted from the 2016 specimen pair or original.

# Algebra

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OxfordAQA 9260 unit 3.2 (specification for first assessment 2018) – the largest unit at 35% of each Extension paper. Core rows A1–A26 cover notation through graphs, equations, inequalities and sequences; the **Extension additions** add subject-twice rearrangement, triple expansions, all quadratic factorisation, fractional powers, quadratic algebraic fractions, formal proof, the full functions toolkit (domain, range, inverse, composite), line equations, exponential and trigonometric graphs, **differentiation** with tangents and stationary points, completing the square, gradients and areas under graphs, variation, every simultaneity pair, quadratic inequalities and quadratic sequences. The 2E specimen samples this unit heavily (algebra is its spine); the unsampled lines are labelled specification originals.

**Must-know for unit 3.2** (NO formulae sheet):

| Family    | Formulae                                                                            |
|-----------|-------------------------------------------------------------------------------------|
| quadratic | $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ; $a(x + b)^2 + c$ gives the turning point |
| line      | $y - y_1 = m(x - x_1)$ ; gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$                 |
| calculus  | $\frac{d}{dx}(kx^n) = knx^{n-1}$ ; tangent: the gradient at the point               |
| nth terms | arithmetic $an + b$ ; quadratic $an^2 + bn + c$                                     |

## Notation, substitution and formulae

Letters express generalised number; substitute numbers into formulae and expressions (positive, negative, fractional). The Extension addition rearranges **complex formulae including when the subject appears twice**: from  $ax + 3 = 5(xt) - 2a$ , collect the subject's terms first –  $ax + 2a = 5xt - 3$ , so  $a(x + 2) = 5xt - 3$  and  $a = \frac{5xt - 3}{x + 2}$ . Check every rearrangement numerically.

## Expanding, factorising and algebraic fractions

Collect like terms; expand single brackets and (Extension) products of two or three binomials:  $(x + 2)(x - 3)(x + 1) = (x^2 - x - 6)(x + 1) = x^3 - 7x - 6$ . Factorise by common factors and (Extension) every quadratic  $ax^2 + bx + c$ , including the difference of two squares:  $2x^2 - 7x - 15 = (2x + 3)(x - 5)$  and  $9x^2 - 25 = (3x - 5)(3x + 5)$ . Index laws run to fractional powers:  $4c^5 \div 3d^3$  stays as  $\frac{4c^5}{3d^3}$  in index-form answers (the 1E specimen's answer shape).

Rational expressions (Extension) add, subtract, multiply and divide linear or quadratic algebraic fractions – factor first, cancel whole factors, common denominators. The 2E

specimen's  $\frac{11m - 14}{m^2 - 3m - 4}$  and  $\frac{2x + 3}{x + 4}$  answers come from this machinery.

## Algebraic proof

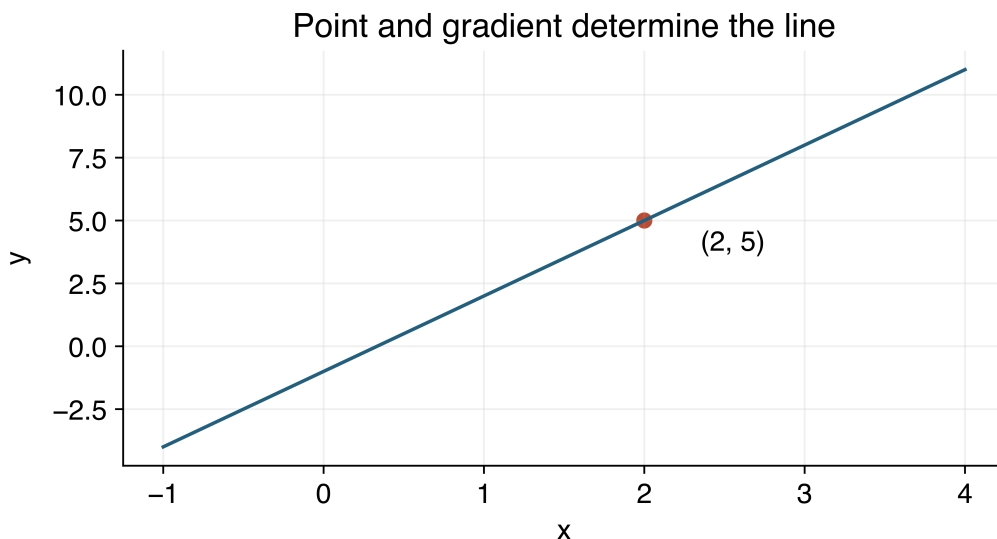
Argue mathematically that identities hold for every value – with the Extension demand for formal **proofs**. Structure: state the algebra, simplify, compare with the claim. Classic (2E specimen family): the expression  $n^2 + n + 9$  for consecutive sums – show  $n(n + 1) + 9$  always sits 9 above a product of consecutive integers. A parity proof:  $(2n + 1)^2 - (2n - 1)^2 = (4n^2 + 4n + 1) - (4n^2 - 4n + 1) = 8n$  – a multiple of 8 for every integer  $n$ . Numeric examples are never a proof.

## Functions

A function maps each input to exactly one output (Core: simple input–output machines; Extension: the formal definition,  $f(x) = \dots$  notation, **domain** and **range**). Composite:  $fg(x) = f(g(x))$ , apply  $g$  first – the 2E specimen builds  $fg(x) = (x - 5)^2 + 1$  and solves  $fg(x) = 5$ :  $(x - 5)^2 = 4$  gives  $x = 3$  (or  $x = 7$ ; check:  $(3 - 5)^2 + 1 = 5$ ). Inverse: swap and rearrange – the 1E specimen's  $f^{-1}(x) = \frac{x}{3}$  comes from  $f(x) = 3x$ .

## Straight-line graphs and coordinates

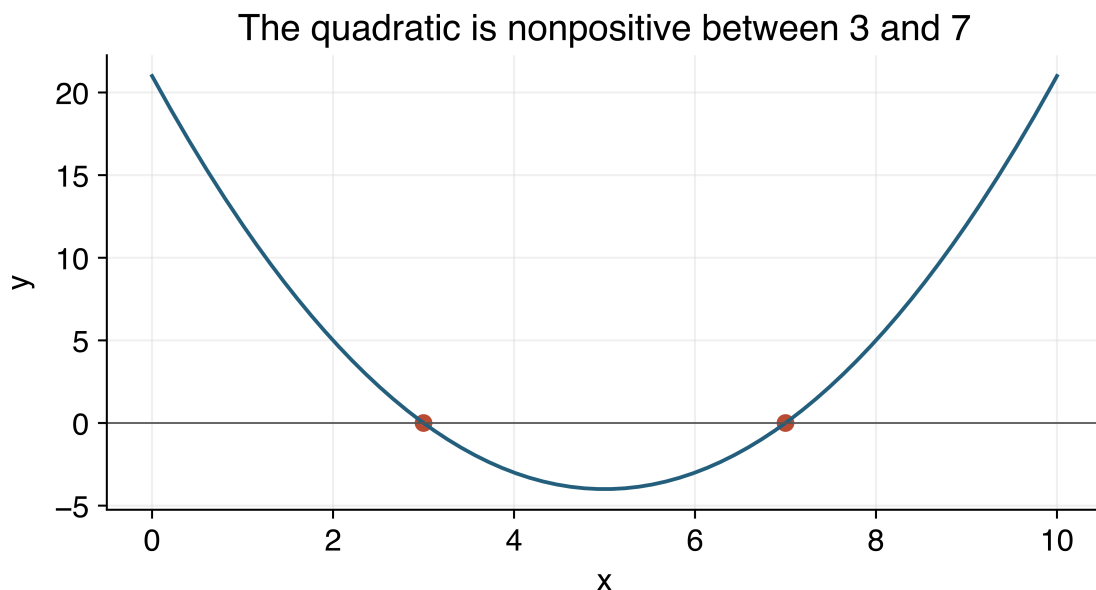
Coordinates in all four quadrants; plot  $y = mx + c$ ; the Extension additions find the equation through two points or through one point with a given gradient, and use  $y - y_1 = m(x - x_1)$ : through (1, 7) and (2, 9),  $m = 2$  and  $y = 2x + 5$ . Parallel lines share  $m$ ; perpendicular gradients multiply to  $-1$  (specification line, not sampled).



*Original native-lesson illustration; labels belong to its worked example.*

## Curves and their graphs

Sketch and interpret linear, quadratic, cubic and reciprocal graphs; the Extension additions add **exponential** functions  $y = k^x$  for positive  $k$  and the trigonometric graphs (degrees) – both specification lines not sampled by the specimens. Identify roots, intercepts and turning points; the Extension addition DEDUCES turning points by **completing the square** :  $y = x^2 - 6x + 5 = (x - 3)^2 - 4$  has its minimum at  $(3, -4)$  (specification line, not sampled). Gradients and areas under graphs (including quadratic and non-linear graphs) are estimated by tangents and trapezia, and – at Extension – computed exactly by differentiation (next section).



*Original native-lesson illustration; labels belong to its worked example.*

## Differentiation

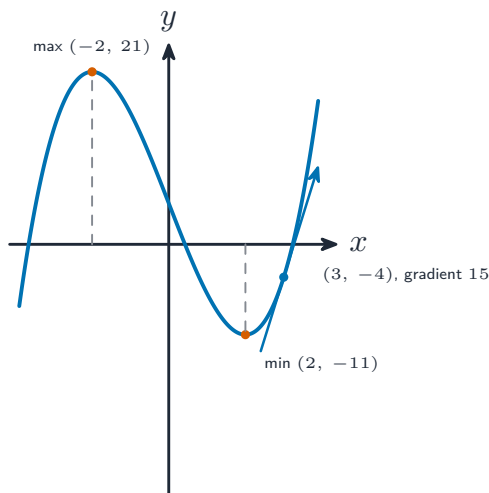
The Extension calculus block: understand and use the **gradient function**  $\frac{dy}{dx}$ , differentiate  $kx^n$  for positive integer  $n$  (and sums), and know that the gradient of a function at a point IS the gradient of the **tangent** there – then work out the tangent's equation.

$$y = x^2 \Rightarrow \frac{dy}{dx} = 2x; \quad \text{at } x = 2 : m = 4, \quad y - 4 = 4(x - 2) \Rightarrow y = 4x - 4$$

The 2E specimen's values (family citation): a curve with  $\frac{dy}{dx} = 15$  at the point carries tangent  $y = 15x - 36$ . Stationary points solve  $\frac{dy}{dx} = 0$ , classified as **maxima, minima**

or **points of inflection** , with curves sketched from the derivatives:

**Worked example (original).**  $y = x^3 - 12x + 5$ :  $\frac{dy}{dx} = 3x^2 - 12 = 0$  gives  $x = \pm 2$ . At  $x = -2$ ,  $y = 21$  (gradient changes + to -: maximum); at  $x = 2$ ,  $y = -11$  (changes - to +: minimum). At  $x = 3$  the gradient is  $3(9) - 12 = 15$  and the tangent is  $y + 4 = 15(x - 3)$ , i.e.  $y = 15x - 49$ .



tangent (blue)  $y = 15x - 49$ ;  
stationary where  $3x^2 - 12 = 0$

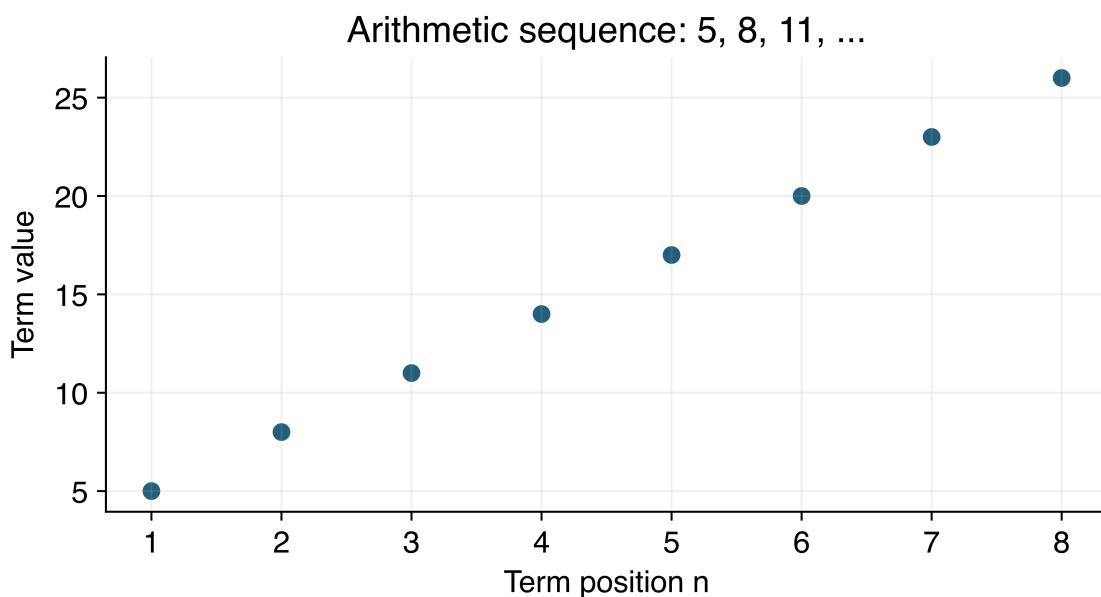
## Equations and inequalities

Solve linear equations (unknown both sides), quadratics by factorising and (Extension) by completing the square and the quadratic formula – the 1E specimen's  $x = -3 \pm \sqrt{7}$  comes from  $x^2 + 6x + 2 = 0$ :  $x = \frac{-6 \pm \sqrt{36-8}}{2}$ . Simultaneous pairs: two linear equations, and (Extension) one linear with one **quadratic** . Variation (Extension): direct  $y = kx^n$  and inverse  $y = \frac{k}{x^n}$  in algebraic terms (specification line, not sampled). Inequalities: linear in one variable, and (Extension) in one or two variables plus **quadratic inequalities** – solve  $x^2 - x - 6 > 0$  by critical values  $x = -2, 3$ : positive quadratic, so  $x < -2$  or  $x > 3$  (specification line, not sampled).

## Sequences

Generate terms from term-to-term and position-to-term rules; use triangular, square and cube number sequences (Core) with the Extension additions covering **quadratic sequences** : second differences are constant, and the  $n$ th term is  $an^2 + bn + c$  with  $a = \frac{1}{2}$ (second difference). For 5, 9, 17, 29, 45, ...: first differences 4, 8, 12, 16, second difference 4, so  $a = 2$ . Take  $2n^2$  (which gives 2, 8, 18, 32, 50) and subtract: the

residuals 3, 1, -1, -3, -5 are linear,  $-2n + 5$ . So the  $n$ th term is  $2n^2 - 2n + 5$  – check  $n = 4$ :  $32 - 8 + 5 = 29$ . ✓



*Original native-lesson illustration; labels belong to its worked example.*

## Checklist

- Generalised notation; substitution; rearrangement with the subject twice (A1–A3, Extension).
- Expanding two or three binomials; factorising all quadratics and the difference of two squares; fractional index answers; quadratic algebraic fractions (A4–A7, Extension; 1E/2E specimens).
- Formal algebraic proof (A8, Extension; 2E  $n^2 + n + 9$  family).
- Functions: definition, domain, range, composite fg, inverse (A9, Extension; 2E  $fg(x) = (x-5)^2 + 1$ , 1E  $f^{-1}(x) = x/3$ ).
- Coordinates; line equations through two points or point + gradient; perpendicular gradients (A10–A11; the latter not sampled).
- Linear, quadratic, cubic, reciprocal, exponential and trigonometric graphs; turning points by completing the square; gradient/area estimates (A12, A16–A17; several lines not sampled).
- Differentiation of  $kx^n$ ; gradient = tangent gradient; tangent equations; stationary points classified with sketches (A13–A15, Extension; 2E tangent family).
- Linear equations; quadratics by all three methods; linear–linear and linear–quadratic simultaneity (A19–A21, Extension).

- Context into algebra (A22, Extension); linear (one/two variable) and quadratic inequalities with solution sets (A23, Extension; not sampled).
- Sequence types;  $n$ th terms linear and quadratic (A24–A26, Extension).

Boundary notes: NO formulae sheet in 9260. Lines marked “not sampled” (perpendicular gradients, exponential/trig sketches, completing-the-square turning points, variation, quadratic and two-variable inequalities) are specification originals; the specimens are Extension-tier only, so Core-tier demand has no local evidence. Worked examples are adapted from the 2016 specimen pair or original.

# Geometry and measures

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OxfordAQA 9260 unit 3.3 (specification Version 5.1) – at 31.5% of the combined specimen marks. Core rows G1–G26 cover angle facts through constructions, mensuration, similarity, trigonometry, transformations, vectors and matrices; the **Extension additions** add the full circle-theorem programme with proof, congruence conditions, bearings, sphere/pyramid/cone/frustum formulae,  $k$ - $k^2$ - $k^3$  similarity, sectors, 3D Pythagoras and trigonometry, the sine/cosine rules with  $\frac{1}{2}ab \sin C$ , combined transformations with fractional/negative scale factors, vector geometry, and the  $2 \times 2$  matrix programme (identity, unit-square transformations, combinations). The 1E specimen is the geometry paper; unsampled lines are labelled specification originals.

**Must-know for unit 3.3** (NO formulae sheet):

| Family    | Formulae                                                                                                                         |
|-----------|----------------------------------------------------------------------------------------------------------------------------------|
| circle    | circumference $2\pi r$ ; area $\pi r^2$ ; arc $\frac{\theta}{360}(2\pi r)$ ; sector $\frac{\theta}{360}\pi r^2$                  |
| solids    | sphere $V = \frac{4}{3}\pi r^3$ , $SA = 4\pi r^2$ ; cone $V = \frac{1}{3}\pi r^2 h$ ; pyramid $V = \frac{1}{3}Ah$<br>(Extension) |
| triangles | $a^2 = b^2 + c^2 - 2bc \cos A$ ; $\frac{a}{\sin A} = \frac{b}{\sin B}$ ; Area = $\frac{1}{2}ab \sin C$                           |

## Angles, polygons and symmetry

Conventional terms: points, lines, vertices, edges, planes; parallel, perpendicular; polygons and regular polygons; the standard triangle labelling. Angle facts: at a point ( $360^\circ$ ), on a line ( $180^\circ$ ), vertically opposite; parallel lines give **alternate**, **corresponding** and **interior** angles (the “co-located” names, never “Z angles”); triangle sum  $180^\circ$  with the isosceles, equilateral and scalene families. Polygon sums: interior  $(n - 2) \times 180^\circ$ , exterior  $360^\circ$ ; for regular polygons also the angle at the centre =  $\frac{360^\circ}{n}$ . The special quadrilaterals (square, rectangle, parallelogram, trapezium, kite, rhombus) carry their side/angle/diagonal properties, and 2D shapes have reflection and rotation symmetry counts (a regular hexagon: 6 lines, order 6).

A length scale factor is squared for area

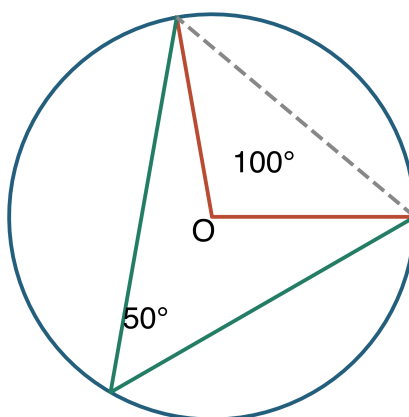


*Original native-lesson illustration; labels belong to its worked example.*

## Congruence, similarity and circle theorems

**Congruence** with the Extension conditions for triangles – SSS, SAS, ASA, RHS – each justified by which three independent measurements pin a triangle. **Similarity** scales lengths by  $k$ : (Extension) areas by  $k^2$ , volumes by  $k^3$ , in every direction.

The centre angle is twice the circumference angle



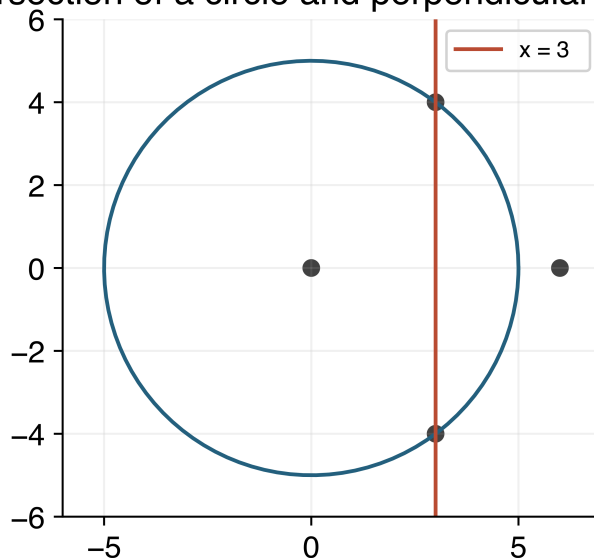
*Original native-lesson illustration; labels belong to its worked example.*

The Extension circle programme: angle at the centre is twice the angle at the circumference; the semicircle angle is  $90^\circ$ ; angles in the same segment are equal; opposite angles of a cyclic quadrilateral sum to  $180^\circ$ ; tangent  $\perp$  radius; tangents from an external point are equal; the perpendicular from the centre bisects a chord; the **alternate segment** theorem. The verified chain: an angle at the circumference of  $35^\circ$  subtends the same arc as a centre angle of  $70^\circ$ , and the angle from the OPPOSITE arc is  $\frac{1}{2}(360^\circ - 70^\circ) = 145^\circ$  – the two circumference angles are opposite angles of a cyclic quadrilateral ( $35^\circ + 145^\circ = 180^\circ$ ) – every step names its theorem (the 1E specimen's circle question carries answers of this family; G9's geometric **proof** demand).

### 3D shapes, constructions and bearings

Faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres; **plans and elevations** (front, side, top views – a specification line not sampled by the specimens). Measure line segments and angles; scale drawings with scale factors; **bearings** : three-figure, clockwise from north (Extension includes the eight compass points) – the 2E specimen's  $252^\circ$  comes from a bearings chain: a return bearing plus an angle at a point.

Intersection of a circle and perpendicular bisector



*Original native-lesson illustration; labels belong to its worked example.*

Ruler-and-compass **constructions** : perpendicular bisector of a segment, perpendicular to a line from/at a point, angle bisector,  $60^\circ$  angle – used to build figures and solve **loci** problems; the perpendicular distance from a point to a line is the shortest. Leave all arcs visible: the arcs are the working.

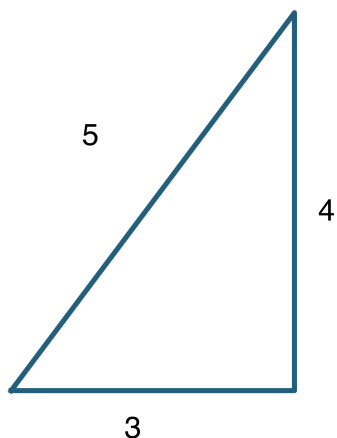
## Units, mensuration and similarity ratios

Standard units with conversions (time 12/24-hour; length, area, volume/capacity, mass; compound units speed and density). Areas: triangle  $\frac{1}{2}bh$ , parallelogram  $bh$ , trapezium  $\frac{1}{2}(a+b)h$ ; volume by  $V = Ah$  (constant cross-section). The circle:  $2\pi r$ ,  $\pi r^2$ ; (Extension) **sectors** arc  $\frac{\theta}{360} \times 2\pi r$  and area  $\frac{\theta}{360} \times \pi r^2$ , plus sphere/pyramid/cone **surface** and **volume** formulae and composite solids including **frustums** (specification lines, several not sampled) – a cone of  $r = 5$ ,  $h = 12$  has slant 13,  $V = 100\pi$  and curved surface  $65\pi$ ; the 1E specimen's cone problem finds a proportion  $p = 15$ . Similar figures (Extension): lengths  $k$  : areas  $k^2$  : volumes  $k^3$  – the 2E specimen's similar solids give  $864 \text{ cm}^3$  from a volume ratio.

## Pythagoras, trigonometry and 3D problems

$a^2 + b^2 = c^2$  with SOH CAH TOA in right-angled triangles – 2D always, 3D figures at Extension: extract the right triangle first (the 1E specimen's 3D trig gives  $62.7^\circ$ ). The sine rule  $\frac{a}{\sin A} = \frac{b}{\sin B}$  and cosine rule  $a^2 = b^2 + c^2 - 2bc \cos A$  cover any triangle (Extension), with Area  $= \frac{1}{2}ab \sin C$  reversible – the 2E specimen's cosine rule gives  $102.6^\circ$ , and  $\cos x = \frac{8}{11}$  yields  $x = 43.3^\circ$ .

Label the known sides before choosing a rule



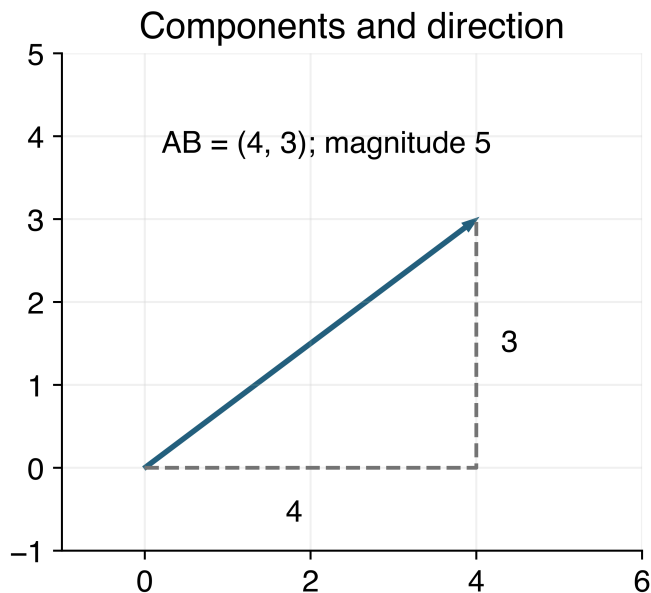
*Original native-lesson illustration; labels belong to its worked example.*

**Worked example (2E cosine-rule family, adapted).** A triangle has  $a = 9$ ,  $b = 11$  and angle  $C = 72^\circ$ .  $c^2 = 81 + 121 - 2(9)(11) \cos 72^\circ = 202 - 198 \times 0.309 = 202 - 61.2 = 140.8$ , so  $c = 11.9$  (3 s.f.). Then the smallest angle sits opposite the shortest side:  $\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{121 + 140.8 - 81}{2 \times 11 \times 11.87} = \frac{180.8}{261.1} = 0.693$ , so  $A = 46.2^\circ$  (keeping  $c$  unrounded) – check  $A + B + C = 180^\circ$  with  $B = 61.8^\circ$ . ✓

## Transformations with matrices and vectors

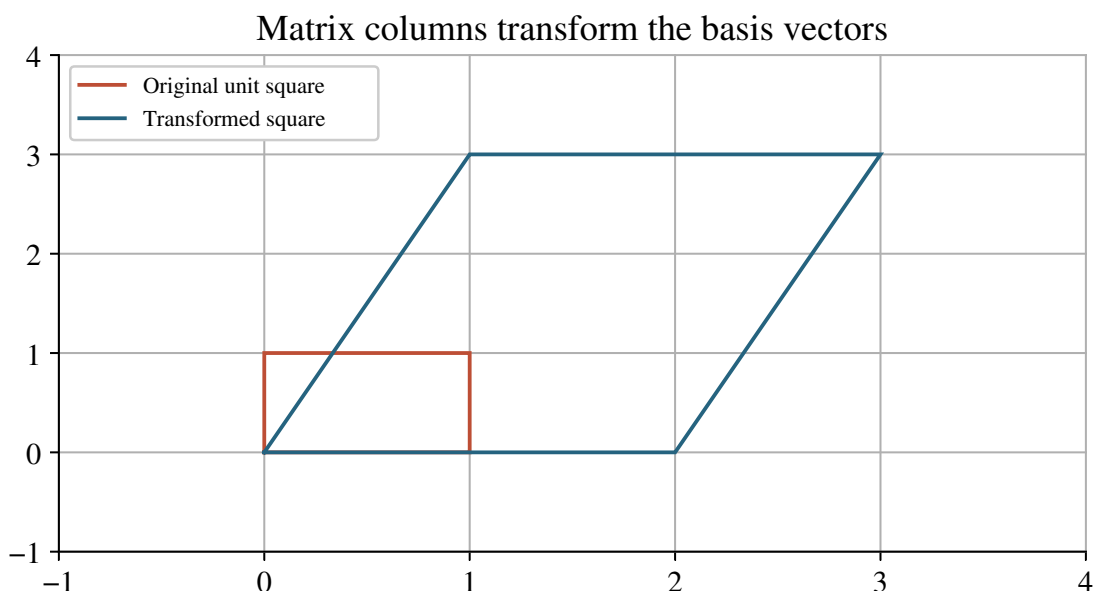
Single rotations, reflections, translations (by a vector) and enlargements (positive factor) – with the preserved properties named; the Extension additions cover **combined transformations** and enlargements by **fractional and negative scale factors**

(an inverted, shrunk image on the opposite side of the centre). **Vectors**  
: notation, sum, difference and scalar multiples (graphically and by components), the commutative and associative laws, and simple 2D geometry – the 2E specimen proves  $CB \parallel MN$  via a  $\frac{1}{2}$  ratio using vectors.



*Original native-lesson illustration; labels belong to its worked example.*

The  $2 \times 2$  **matrix** programme (Extension): multiplication ( $2 \times 2$  by  $2 \times 2$ , by  $2 \times 1$ , scalar), the **identity matrix**  $I$ ; transformations of the UNIT SQUARE – rotations of  $90^\circ/180^\circ/270^\circ$  about the origin, reflections in  $x = 0$ ,  $y = 0$ ,  $y = x$ ,  $y = -x$ , enlargements centred on the origin – represented by matrices such as  $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$  (reflection in  $y = -x$ , the 1E specimen's matrix); combined transformations by matrix multiplication, in the order the transformations apply.



*Original native-lesson illustration; labels belong to its worked example.*

**Worked example (1E matrix-solve family, adapted).** Solve  $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$ : the equations are  $3x + y = 10$  and  $x + 2y = 5$ . From the first,  $y = 10 - 3x$ ; substituting:  $x + 2(10 - 3x) = 5$ , so  $-5x = -15$ , giving  $x = 3$  and  $y = 1$ . Check:  $3(3) + 1 = 10$ . ✓ The 1E specimen's own pairs  $(6, -\frac{1}{2})$  and  $(-2, -\frac{5}{2})$  come from the same method on a second, linked system.

## Checklist

- Conventional terms and triangle notation; angle facts; parallel-line angles; polygon interior/exterior/centre sums; quadrilateral properties; symmetry counts (G1–G6).
- Congruence with SSS/SAS/ASA/RHS; similarity with  $k$ ,  $k^2$ ,  $k^3$  ratios (G7, G17, Extension).
- Circle language and the full theorem bank with proof-style reasons (G8–G9, Extension;  $35^\circ \rightarrow 70^\circ \rightarrow 145^\circ$  verified chain).
- 3D shape properties; plans and elevations (not sampled); measuring, scale factors, bearings (G10–G12).
- Constructions (four ruler-and-compass types) with loci; shortest distance property (G13).
- Units and compound measures; area and  $V = Ah$  formulae; circle formulae; sectors; sphere/pyramid/cone/frustum surface and volume; composite shapes (G14–G16, Extension).

- Pythagoras and trigonometry in 2D and 3D; sine/cosine rules and  $\frac{1}{2}ab \sin C$  (G19–G20, Extension; 1E  $62.7^\circ$ , 2E  $102.6^\circ$  and  $\cos x = 8/11$ ).
- Transformations including combined and fractional/negative enlargements (G21, Extension).
- Vector notation and geometry (G22; 2E  $CB \parallel MN$  proof).
- Matrix multiplication, identity, unit-square transformations and combinations (G23–G26, Extension; 1E matrices).

Boundary notes: NO formulae sheet in 9260. Lines marked “not sampled” (plans and elevations, parts of the solids programme) are specification originals; the specimens are Extension-tier only. Worked examples are adapted from the 2016 specimen pair or original.

# Statistics and probability

OxfordAQA · International GCSE · Mathematics

OxfordAQA 9260 unit 3.4 (specification Version 5.1) – the smallest unit at 13% of the combined specimen marks. Core rows S1–S5 cover data types, reading tables, two-way tables, the full chart bank and the basic averages; the **Extension additions** add histograms with UNEQUAL class intervals, cumulative frequency diagrams, box plots, and quartiles with the inter-quartile range and percentiles. The 1E specimen carries a histogram and a cumulative-frequency correction to upper boundaries; the 2E specimen’s statistics questions include the 40-men and 38% reads. Lines the specimens never sample are labelled specification originals.

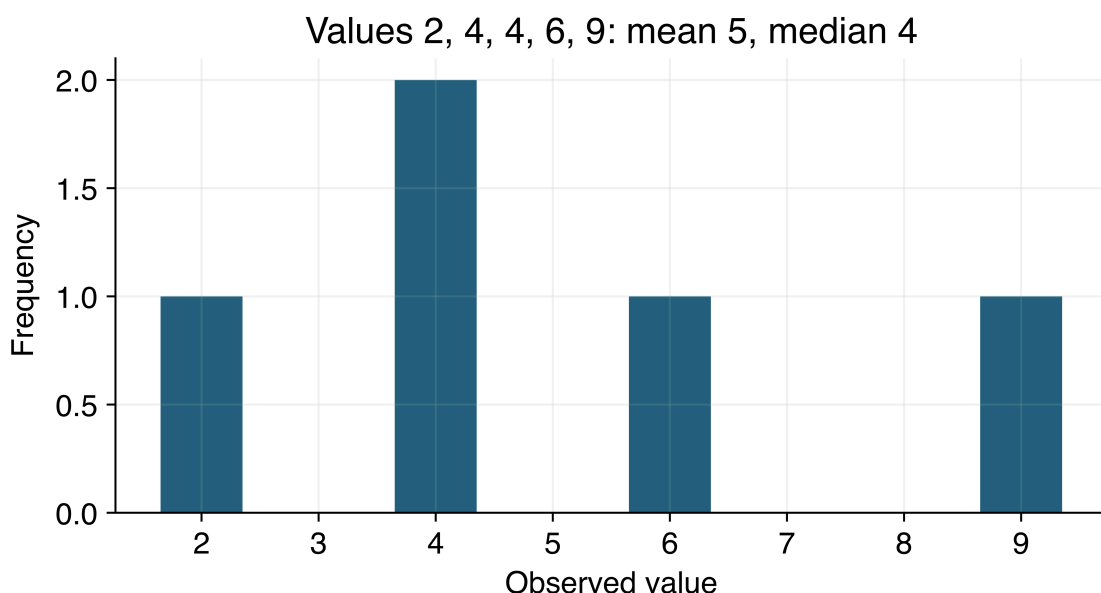
## Must-know for unit 3.4:

| Family               | Formulae                                                                                             |
|----------------------|------------------------------------------------------------------------------------------------------|
| histogram            | frequency density = $\frac{\text{frequency}}{\text{class width}}$ – bar AREA is the frequency        |
| cumulative frequency | plot at the UPPER class boundary; median at $\frac{n}{2}$ , quartiles at $\frac{n}{4}, \frac{3n}{4}$ |
| averages             | mean = $\frac{\sum fx}{\sum f}$ (grouped: midpoints); IQR = $Q_3 - Q_1$                              |

## Data types, tables and charts

**Qualitative** data categorises (favourite fruit); **discrete** data counts (number of goals); **continuous** data measures (height), grouped or ungrouped. Extract data from printed tables and lists – read the row and column headers first, and quote units. **Two-way tables** cross-tabulate two variables; every row and column total checks the interior cells: 30 students, 18 cricket, 15 tennis and 7 both fills a 2×2 whose cells must sum to 30.

The Core chart bank: scatter graphs, stem-and-leaf, tally charts, pictograms, bar charts, dual and composite bar charts, pie charts, line graphs, frequency polygons and histograms with EQUAL class intervals. Choose by data type – categories take bars, comparisons take dual bars, parts-of-a-whole take pie charts, trends take line graphs, correlation takes scatter graphs.



*Original native-lesson illustration; labels belong to its worked example.*

## Extension diagrams: unequal histograms, CF and box plots

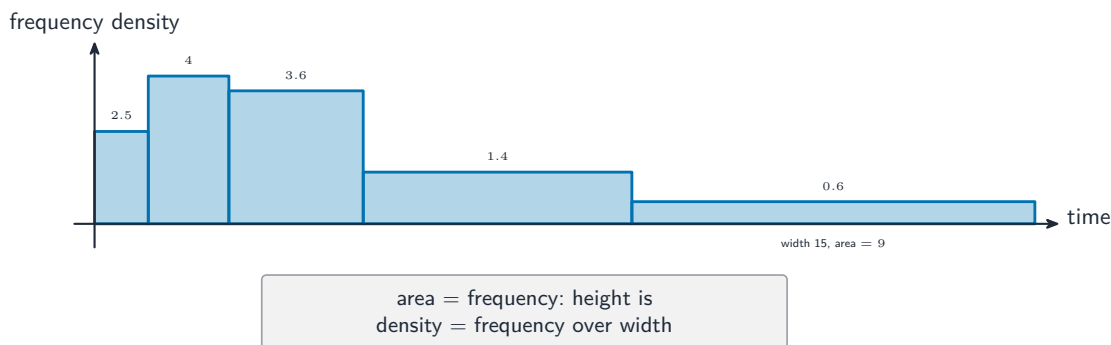
A **histogram** with **unequal class intervals** draws bar HEIGHT as the **frequency density** = frequency over class width – the AREA is the frequency (classes of widths 2, 3, 5, 10, 15 holding 5, 12, 18, 14, 9 give heights 2.5, 4, 3.6, 1.4, 0.6). A **cumulative frequency diagram** plots each running total at its UPPER class boundary as a smooth increasing curve (the 1E specimen's correction is exactly this upper-boundary point placement); the median reads at  $\frac{n}{2}$ , quartiles at  $\frac{n}{4}$  and  $\frac{3n}{4}$ . A **box plot** (five-number summary: minimum,  $Q_1$ , median,  $Q_3$ , maximum) shows spread at a glance – the box spans the IQR, the whiskers the range, outliers by judgement.

## Averages, quartiles and percentiles

The **mean** divides the total by the count (grouped data multiplies each MID-POINT by its frequency first); the **median** orders and takes the middle; the **mode** is most frequent and the **modal class** is the class with the highest frequency DENSITY when widths differ. The Extension additions add **quartiles** and the **inter-quartile range** =  $Q_3 - Q_1$  (the median of the lower and upper halves), and **percentiles** read like the median at any percentage – the  $p$ th percentile is the value  $\frac{p}{100}n$  of the way up the ordered data or the cumulative-frequency curve.

**Worked example (original).** 60 workers' times have cumulative frequencies 6, 17, 27, 42, 60 at upper boundaries 10, 15, 20, 25, 30 (hours). The median reads at worker

30: between 20 (27) and 25 (42), so  $20 + 5 \times \frac{30-27}{42-27} = 21$  hours.  $Q_1$  at worker 15:  $10 + 5 \times \frac{15-6}{17-6} = 14.1$ ;  $Q_3$  at worker 45:  $25 + 5 \times \frac{45-42}{60-42} = 25.8$ ; IQR  $\approx 11.7$  hours. Draw the reading lines – the marks are for the reads.



## Checklist

- Data types qualitative/discrete/continuous; grouped and ungrouped (S1).
- Extract from printed tables and lists, quoting units (S2).
- Design and use two-way tables with row/column checks (S3).
- The Core chart bank, chosen by data type (S4).
- Unequal-interval histograms by frequency density (area is frequency); cumulative frequency diagrams at UPPER boundaries with median/quartile reads; box plots (S4 Extension; 1E histogram and CF-boundary anchors).
- Mean (grouped via midpoints), median, mode and modal class (S5).
- Quartiles, inter-quartile range and percentiles from data or the curve (S5 Extension).

Boundary notes: probability vocabulary sits in this specification's Number/Algebra work and earlier units' sets; the AO split is asserted by the specification. Lines not sampled by the specimens (stem-and-leaf, pie charts and several chart types on the Extension papers) are taught as specification originals; the specimens are Extension-tier only, so Core-tier demand has no local evidence. Worked examples are adapted from the 2016 specimen pair or original.