

Geometry and measures

OxfordAQA · International GCSE · Mathematics

OxfordAQA 9260 unit 3.3 (specification Version 5.1) – at 31.5% of the combined specimen marks. Core rows G1–G26 cover angle facts through constructions, mensuration, similarity, trigonometry, transformations, vectors and matrices; the **Extension additions** add the full circle-theorem programme with proof, congruence conditions, bearings, sphere/pyramid/cone/frustum formulae, k - k^2 - k^3 similarity, sectors, 3D Pythagoras and trigonometry, the sine/cosine rules with $\frac{1}{2}ab \sin C$, combined transformations with fractional/negative scale factors, vector geometry, and the 2×2 matrix programme (identity, unit-square transformations, combinations). The 1E specimen is the geometry paper; unsampled lines are labelled specification originals.

Must-know for unit 3.3 (NO formulae sheet):

Family	Formulae
circle	circumference $2\pi r$; area πr^2 ; arc $\frac{\theta}{360}(2\pi r)$; sector $\frac{\theta}{360}\pi r^2$
solids	sphere $V = \frac{4}{3}\pi r^3$, $SA = 4\pi r^2$; cone $V = \frac{1}{3}\pi r^2 h$; pyramid $V = \frac{1}{3}Ah$ (Extension)
triangles	$a^2 = b^2 + c^2 - 2bc \cos A$; $\frac{a}{\sin A} = \frac{b}{\sin B}$; Area = $\frac{1}{2}ab \sin C$

Angles, polygons and symmetry

Conventional terms: points, lines, vertices, edges, planes; parallel, perpendicular; polygons and regular polygons; the standard triangle labelling. Angle facts: at a point (360°), on a line (180°), vertically opposite; parallel lines give **alternate**, **corresponding** and **interior** angles (the “co-located” names, never “Z angles”); triangle sum 180° with the isosceles, equilateral and scalene families. Polygon sums: interior $(n - 2) \times 180^\circ$, exterior 360° ; for regular polygons also the angle at the centre = $\frac{360^\circ}{n}$. The special quadrilaterals (square, rectangle, parallelogram, trapezium, kite, rhombus) carry their side/angle/diagonal properties, and 2D shapes have reflection and rotation symmetry counts (a regular hexagon: 6 lines, order 6).

A length scale factor is squared for area

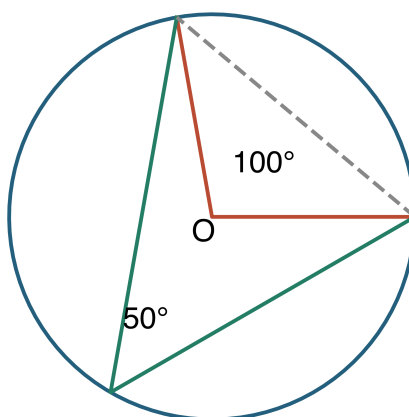


Original native-lesson illustration; labels belong to its worked example.

Congruence, similarity and circle theorems

Congruence with the Extension conditions for triangles – SSS, SAS, ASA, RHS – each justified by which three independent measurements pin a triangle. **Similarity** scales lengths by k : (Extension) areas by k^2 , volumes by k^3 , in every direction.

The centre angle is twice the circumference angle



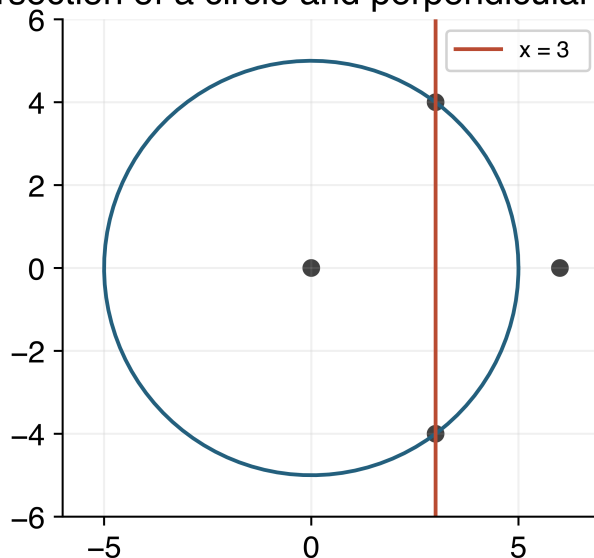
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The Extension circle programme: angle at the centre is twice the angle at the circumference; the semicircle angle is 90° ; angles in the same segment are equal; opposite angles of a cyclic quadrilateral sum to 180° ; tangent $\perp$ radius; tangents from an external point are equal; the perpendicular from the centre bisects a chord; the **alternate segment** theorem. The verified chain: an angle at the circumference of 35° subtends the same arc as a centre angle of 70° , and the angle from the OPPOSITE arc is $\frac{1}{2}(360^\circ - 70^\circ) = 145^\circ$ – the two circumference angles are opposite angles of a cyclic quadrilateral ($35^\circ + 145^\circ = 180^\circ$) – every step names its theorem (the 1E specimen's circle question carries answers of this family; G9's geometric **proof** demand).

3D shapes, constructions and bearings

Faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres; **plans and elevations** (front, side, top views – a specification line not sampled by the specimens). Measure line segments and angles; scale drawings with scale factors; **bearings** : three-figure, clockwise from north (Extension includes the eight compass points) – the 2E specimen's 252° comes from a bearings chain: a return bearing plus an angle at a point.

Intersection of a circle and perpendicular bisector



Original native-lesson illustration; labels belong to its worked example.

Ruler-and-compass **constructions** : perpendicular bisector of a segment, perpendicular to a line from/at a point, angle bisector, 60° angle – used to build figures and solve **loci** problems; the perpendicular distance from a point to a line is the shortest. Leave all arcs visible: the arcs are the working.

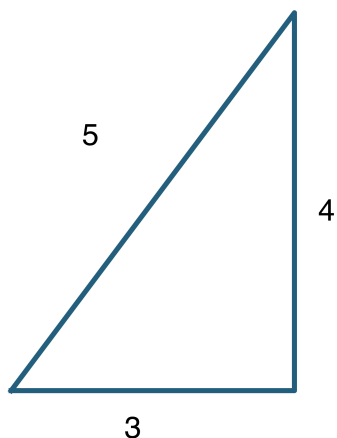
Units, mensuration and similarity ratios

Standard units with conversions (time 12/24-hour; length, area, volume/capacity, mass; compound units speed and density). Areas: triangle $\frac{1}{2}bh$, parallelogram bh , trapezium $\frac{1}{2}(a+b)h$; volume by $V = Ah$ (constant cross-section). The circle: $2\pi r$, πr^2 ; (Extension) **sectors** arc $\frac{\theta}{360} \times 2\pi r$ and area $\frac{\theta}{360} \times \pi r^2$, plus sphere/pyramid/cone **surface** and **volume** formulae and composite solids including **frustums** (specification lines, several not sampled) – a cone of $r = 5$, $h = 12$ has slant 13, $V = 100\pi$ and curved surface 65π ; the 1E specimen's cone problem finds a proportion $p = 15$. Similar figures (Extension): lengths k : areas k^2 : volumes k^3 – the 2E specimen's similar solids give 864 cm^3 from a volume ratio.

Pythagoras, trigonometry and 3D problems

$a^2 + b^2 = c^2$ with SOH CAH TOA in right-angled triangles – 2D always, 3D figures at Extension: extract the right triangle first (the 1E specimen's 3D trig gives 62.7°). The sine rule $\frac{a}{\sin A} = \frac{b}{\sin B}$ and cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$ cover any triangle (Extension), with Area $= \frac{1}{2}ab \sin C$ reversible – the 2E specimen's cosine rule gives 102.6° , and $\cos x = \frac{8}{11}$ yields $x = 43.3^\circ$.

Label the known sides before choosing a rule



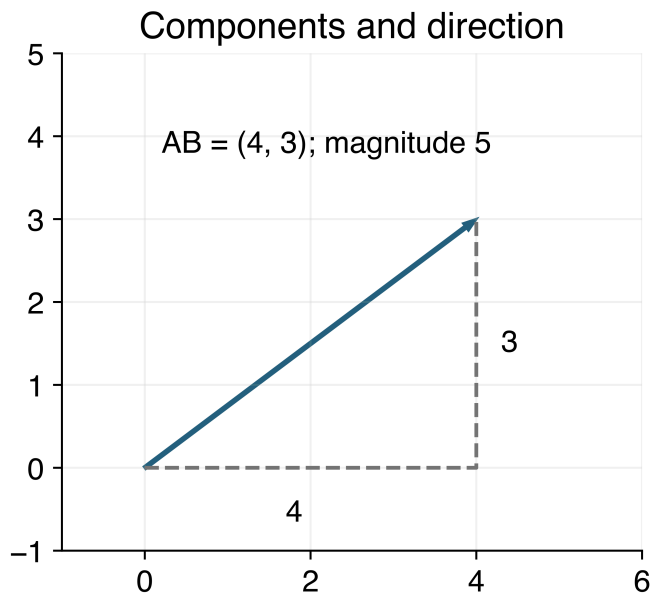
Original native-lesson illustration; labels belong to its worked example.

Worked example (2E cosine-rule family, adapted). A triangle has $a = 9$, $b = 11$ and angle $C = 72^\circ$. $c^2 = 81 + 121 - 2(9)(11) \cos 72^\circ = 202 - 198 \times 0.309 = 202 - 61.2 = 140.8$, so $c = 11.9$ (3 s.f.). Then the smallest angle sits opposite the shortest side: $\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{121 + 140.8 - 81}{2 \times 11 \times 11.87} = \frac{180.8}{261.1} = 0.693$, so $A = 46.2^\circ$ (keeping c unrounded) – check $A + B + C = 180^\circ$ with $B = 61.8^\circ$. ✓

Transformations with matrices and vectors

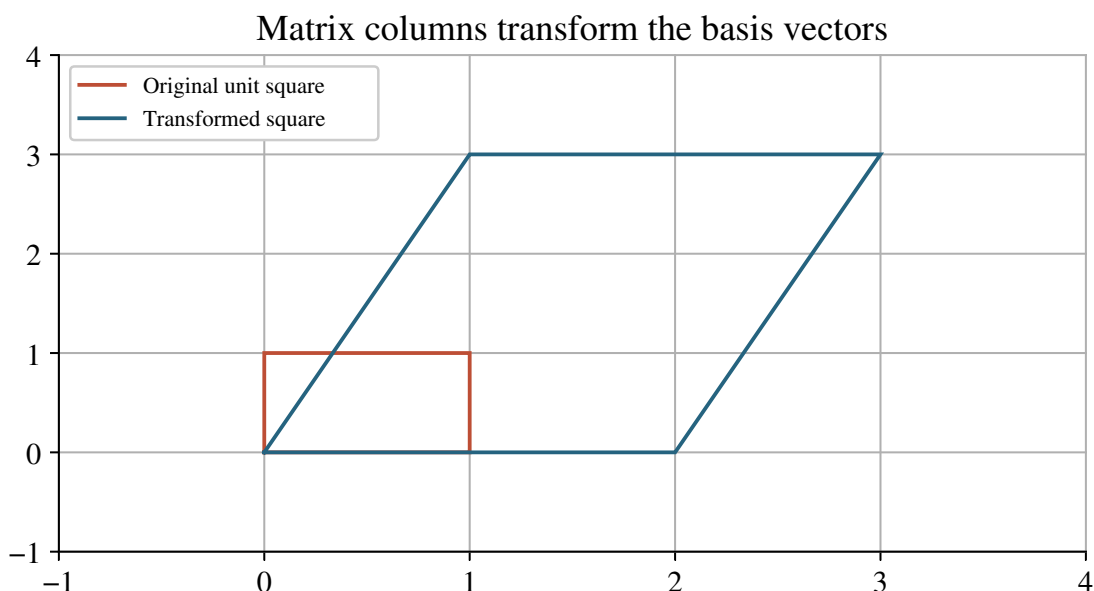
Single rotations, reflections, translations (by a vector) and enlargements (positive factor) – with the preserved properties named; the Extension additions cover **combined transformations** and enlargements by **fractional and negative scale factors**

(an inverted, shrunk image on the opposite side of the centre). **Vectors**
: notation, sum, difference and scalar multiples (graphically and by components), the commutative and associative laws, and simple 2D geometry – the 2E specimen proves $CB \parallel MN$ via a $\frac{1}{2}$ ratio using vectors.



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The 2×2 **matrix** programme (Extension): multiplication (2×2 by 2×2 , by 2×1 , scalar), the **identity matrix** I ; transformations of the UNIT SQUARE – rotations of $90^\circ/180^\circ/270^\circ$ about the origin, reflections in $x = 0$, $y = 0$, $y = x$, $y = -x$, enlargements centred on the origin – represented by matrices such as $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ (reflection in $y = -x$, the 1E specimen's matrix); combined transformations by matrix multiplication, in the order the transformations apply.



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Worked example (1E matrix-solve family, adapted). Solve $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$: the equations are $3x + y = 10$ and $x + 2y = 5$. From the first, $y = 10 - 3x$; substituting: $x + 2(10 - 3x) = 5$, so $-5x = -15$, giving $x = 3$ and $y = 1$. Check: $3(3) + 1 = 10$. ✓ The 1E specimen's own pairs $(6, -\frac{1}{2})$ and $(-2, -\frac{5}{2})$ come from the same method on a second, linked system.

Checklist

- Conventional terms and triangle notation; angle facts; parallel-line angles; polygon interior/exterior/centre sums; quadrilateral properties; symmetry counts (G1–G6).
- Congruence with SSS/SAS/ASA/RHS; similarity with k , k^2 , k^3 ratios (G7, G17, Extension).
- Circle language and the full theorem bank with proof-style reasons (G8–G9, Extension; $35^\circ \rightarrow 70^\circ \rightarrow 145^\circ$ verified chain).
- 3D shape properties; plans and elevations (not sampled); measuring, scale factors, bearings (G10–G12).
- Constructions (four ruler-and-compass types) with loci; shortest distance property (G13).
- Units and compound measures; area and $V = Ah$ formulae; circle formulae; sectors; sphere/pyramid/cone/frustum surface and volume; composite shapes (G14–G16, Extension).

- Pythagoras and trigonometry in 2D and 3D; sine/cosine rules and $\frac{1}{2}ab \sin C$ (G19–G20, Extension; 1E 62.7° , 2E 102.6° and $\cos x = 8/11$).
- Transformations including combined and fractional/negative enlargements (G21, Extension).
- Vector notation and geometry (G22; 2E $CB \parallel MN$ proof).
- Matrix multiplication, identity, unit-square transformations and combinations (G23–G26, Extension; 1E matrices).

Boundary notes: NO formulae sheet in 9260. Lines marked “not sampled” (plans and elevations, parts of the solids programme) are specification originals; the specimens are Extension-tier only. Worked examples are adapted from the 2016 specimen pair or original.