

Algebra

OxfordAQA · International GCSE · Mathematics

OxfordAQA 9260 unit 3.2 (specification for first assessment 2018) – the largest unit at 35% of each Extension paper. Core rows A1–A26 cover notation through graphs, equations, inequalities and sequences; the **Extension additions** add subject-twice rearrangement, triple expansions, all quadratic factorisation, fractional powers, quadratic algebraic fractions, formal proof, the full functions toolkit (domain, range, inverse, composite), line equations, exponential and trigonometric graphs, **differentiation** with tangents and stationary points, completing the square, gradients and areas under graphs, variation, every simultaneity pair, quadratic inequalities and quadratic sequences. The 2E specimen samples this unit heavily (algebra is its spine); the unsampled lines are labelled specification originals.

Must-know for unit 3.2 (NO formulae sheet):

Family	Formulae
quadratic	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $a(x + b)^2 + c$ gives the turning point
line	$y - y_1 = m(x - x_1)$; gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$
calculus	$\frac{d}{dx}(kx^n) = knx^{n-1}$; tangent: the gradient at the point
nth terms	arithmetic $an + b$; quadratic $an^2 + bn + c$

Notation, substitution and formulae

Letters express generalised number; substitute numbers into formulae and expressions (positive, negative, fractional). The Extension addition rearranges **complex formulae including when the subject appears twice**: from $ax + 3 = 5(xt) - 2a$, collect the subject's terms first – $ax + 2a = 5xt - 3$, so $a(x + 2) = 5xt - 3$ and $a = \frac{5xt - 3}{x + 2}$. Check every rearrangement numerically.

Expanding, factorising and algebraic fractions

Collect like terms; expand single brackets and (Extension) products of two or three binomials: $(x + 2)(x - 3)(x + 1) = (x^2 - x - 6)(x + 1) = x^3 - 7x - 6$. Factorise by common factors and (Extension) every quadratic $ax^2 + bx + c$, including the difference of two squares: $2x^2 - 7x - 15 = (2x + 3)(x - 5)$ and $9x^2 - 25 = (3x - 5)(3x + 5)$. Index laws run to fractional powers: $4c^5 \div 3d^3$ stays as $\frac{4c^5}{3d^3}$ in index-form answers (the 1E specimen's answer shape).

Rational expressions (Extension) add, subtract, multiply and divide linear or quadratic algebraic fractions – factor first, cancel whole factors, common denominators. The 2E

specimen's $\frac{11m - 14}{m^2 - 3m - 4}$ and $\frac{2x + 3}{x + 4}$ answers come from this machinery.

Algebraic proof

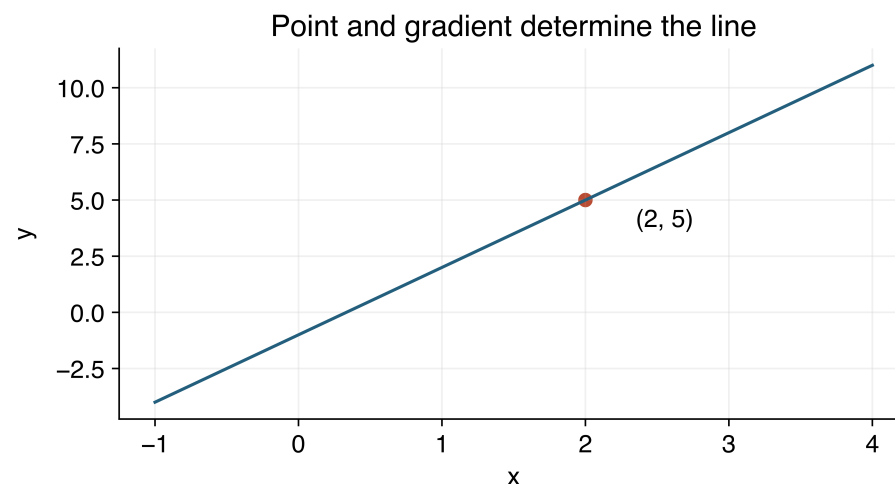
Argue mathematically that identities hold for every value – with the Extension demand for formal **proofs**. Structure: state the algebra, simplify, compare with the claim. Classic (2E specimen family): the expression $n^2 + n + 9$ for consecutive sums – show $n(n + 1) + 9$ always sits 9 above a product of consecutive integers. A parity proof: $(2n + 1)^2 - (2n - 1)^2 = (4n^2 + 4n + 1) - (4n^2 - 4n + 1) = 8n$ – a multiple of 8 for every integer n . Numeric examples are never a proof.

Functions

A function maps each input to exactly one output (Core: simple input–output machines; Extension: the formal definition, $f(x) = \dots$ notation, **domain** and **range**). Composite: $fg(x) = f(g(x))$, apply g first – the 2E specimen builds $fg(x) = (x - 5)^2 + 1$ and solves $fg(x) = 5$: $(x - 5)^2 = 4$ gives $x = 3$ (or $x = 7$; check: $(3 - 5)^2 + 1 = 5$). Inverse: swap and rearrange – the 1E specimen's $f^{-1}(x) = \frac{x}{3}$ comes from $f(x) = 3x$.

Straight-line graphs and coordinates

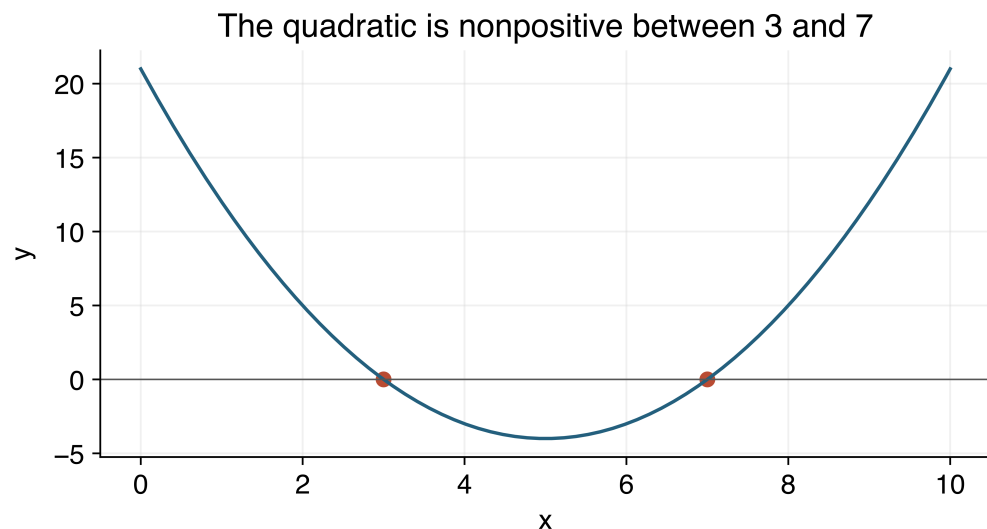
Coordinates in all four quadrants; plot $y = mx + c$; the Extension additions find the equation through two points or through one point with a given gradient, and use $y - y_1 = m(x - x_1)$: through (1, 7) and (2, 9), $m = 2$ and $y = 2x + 5$. Parallel lines share m ; perpendicular gradients multiply to -1 (specification line, not sampled).



Original native-lesson illustration; labels belong to its worked example.

Curves and their graphs

Sketch and interpret linear, quadratic, cubic and reciprocal graphs; the Extension additions add **exponential** functions $y = k^x$ for positive k and the trigonometric graphs (degrees) – both specification lines not sampled by the specimens. Identify roots, intercepts and turning points; the Extension addition DEDUCES turning points by **completing the square** : $y = x^2 - 6x + 5 = (x - 3)^2 - 4$ has its minimum at $(3, -4)$ (specification line, not sampled). Gradients and areas under graphs (including quadratic and non-linear graphs) are estimated by tangents and trapezia, and – at Extension – computed exactly by differentiation (next section).



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Differentiation

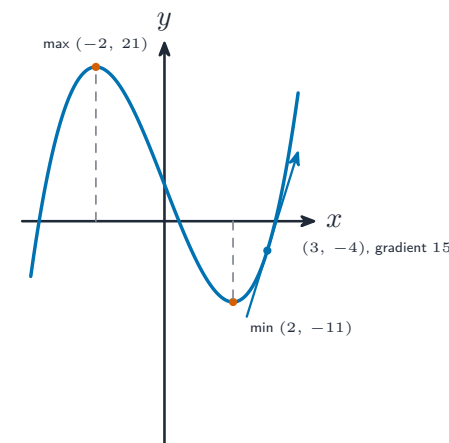
The Extension calculus block: understand and use the **gradient function** $\frac{dy}{dx}$, differentiate kx^n for positive integer n (and sums), and know that the gradient of a function at a point IS the gradient of the **tangent** there – then work out the tangent's equation.

$$y = x^2 \Rightarrow \frac{dy}{dx} = 2x; \quad \text{at } x = 2: m = 4, \quad y - 4 = 4(x - 2) \Rightarrow y = 4x - 4$$

The 2E specimen's values (family citation): a curve with $\frac{dy}{dx} = 15$ at the point carries tangent $y = 15x - 36$. Stationary points solve $\frac{dy}{dx} = 0$, classified as **maxima**, **minima**

or **points of inflection** , with curves sketched from the derivatives:

Worked example (original). $y = x^3 - 12x + 5$: $\frac{dy}{dx} = 3x^2 - 12 = 0$ gives $x = \pm 2$. At $x = -2$, $y = 21$ (gradient changes + to -: maximum); at $x = 2$, $y = -11$ (changes - to +: minimum). At $x = 3$ the gradient is $3(9) - 12 = 15$ and the tangent is $y + 4 = 15(x - 3)$, i.e. $y = 15x - 49$.



tangent (blue) $y = 15x - 49$;
stationary where $3x^2 - 12 = 0$

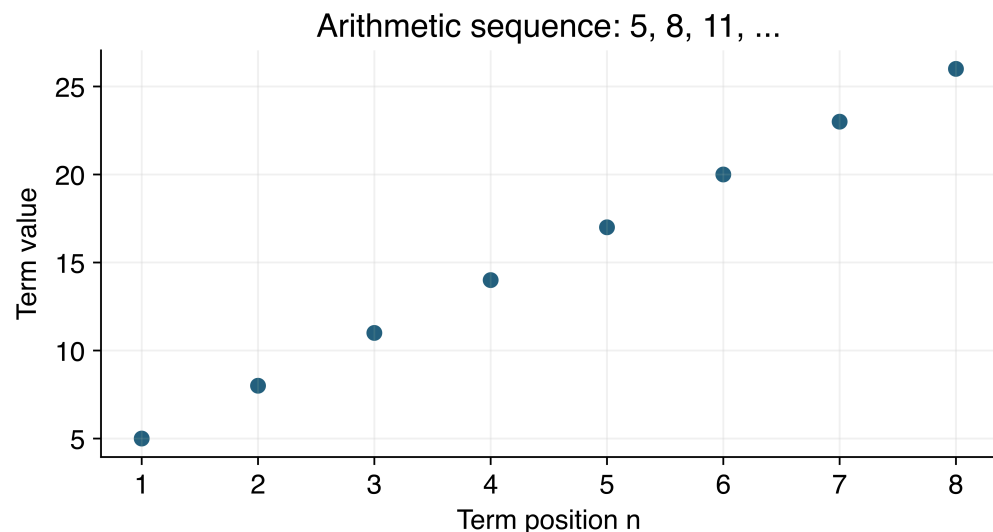
Equations and inequalities

Solve linear equations (unknown both sides), quadratics by factorising and (Extension) by completing the square and the quadratic formula – the 1E specimen's $x = -3 \pm \sqrt{7}$ comes from $x^2 + 6x + 2 = 0$: $x = \frac{-6 \pm \sqrt{36 - 8}}{2}$. Simultaneous pairs: two linear equations, and (Extension) one linear with one **quadratic** . Variation (Extension): direct $y = kx^n$ and inverse $y = \frac{k}{x^n}$ in algebraic terms (specification line, not sampled). Inequalities: linear in one variable, and (Extension) in one or two variables plus **quadratic inequalities** – solve $x^2 - x - 6 > 0$ by critical values $x = -2, 3$: positive quadratic, so $x < -2$ or $x > 3$ (specification line, not sampled).

Sequences

Generate terms from term-to-term and position-to-term rules; use triangular, square and cube number sequences (Core) with the Extension additions covering **quadratic sequences** : second differences are constant, and the n th term is $an^2 + bn + c$ with $a = \frac{1}{2}$ (second difference). For 5, 9, 17, 29, 45, ...: first differences 4, 8, 12, 16, second difference 4, so $a = 2$. Take $2n^2$ (which gives 2, 8, 18, 32, 50) and subtract: the

residuals 3, 1, -1, -3, -5 are linear, $-2n + 5$. So the n th term is $2n^2 - 2n + 5$ – check $n = 4$: $32 - 8 + 5 = 29$. ✓



Original native-lesson illustration; labels belong to its worked example.

Checklist

- Generalised notation; substitution; rearrangement with the subject twice (A1–A3, Extension).
- Expanding two or three binomials; factorising all quadratics and the difference of two squares; fractional index answers; quadratic algebraic fractions (A4–A7, Extension; 1E/2E specimens).
- Formal algebraic proof (A8, Extension; 2E $n^2 + n + 9$ family).
- Functions: definition, domain, range, composite fg, inverse (A9, Extension; 2E $fg(x) = (x-5)^2 + 1$, 1E $f^{-1}(x) = x/3$).
- Coordinates; line equations through two points or point + gradient; perpendicular gradients (A10–A11; the latter not sampled).
- Linear, quadratic, cubic, reciprocal, exponential and trigonometric graphs; turning points by completing the square; gradient/area estimates (A12, A16–A17; several lines not sampled).
- Differentiation of kx^n ; gradient = tangent gradient; tangent equations; stationary points classified with sketches (A13–A15, Extension; 2E tangent family).
- Linear equations; quadratics by all three methods; linear–linear and linear–quadratic simultaneity (A19–A21, Extension).

- Context into algebra (A22, Extension); linear (one/two variable) and quadratic inequalities with solution sets (A23, Extension; not sampled).
- Sequence types; n th terms linear and quadratic (A24–A26, Extension).

Boundary notes: NO formulae sheet in 9260. Lines marked “not sampled” (perpendicular gradients, exponential/trig sketches, completing-the-square turning points, variation, quadratic and two-variable inequalities) are specification originals; the specimens are Extension-tier only, so Core-tier demand has no local evidence. Worked examples are adapted from the 2016 specimen pair or original.