

3.4 Trigonometry in 2D and 3D

Name: _____ Class: _____ Date: _____ Total: 23 marks

Trigonometry in 2D and 3D

Objective

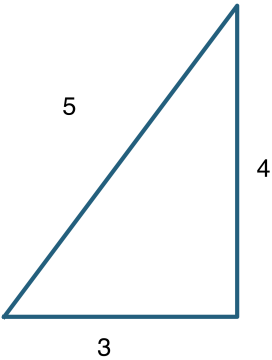
Use Pythagoras and SOH CAH TOA in right triangles (2D and 3D); the sine and cosine rules with the $\frac{1}{2}ab \sin C$ area formula in any triangle (rows G19–G20 with the Extension additions).

1 Key knowledge and worked examples

Right triangles. $a^2 + b^2 = c^2$; $\sin \theta = \frac{\text{opp}}{\text{hyp}}$, $\cos \theta = \frac{\text{adj}}{\text{hyp}}$, $\tan \theta = \frac{\text{opp}}{\text{adj}}$ – in 3D (Extension) extract the right triangle first: the 1E specimen’s chain ends at 62.7° .

Any triangle. Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B}$; cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$; Area $= \frac{1}{2}ab \sin C$ – the 2E specimen gives 102.6° by the cosine rule and $x = 43.3^\circ$ from $\cos x = \frac{8}{11}$.

Label the known sides before choosing a rule



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 A right triangle has legs 5 and 12. Find the hypotenuse. [2]

2.2 Find θ where the opposite side is 7 and the hypotenuse is 9, to 1 d.p. [2]

2.3 A triangle has $a = 8$, $b = 11$, $C = 60^\circ$. Find c . [3]

2.4 $c^2 = 81 + 121 - 198 \cos 72^\circ = 202 - 61.2 = 140.8$: $c = 11.9$; smallest angle opposite the shortest side: $\cos A = \frac{121+140.8-81}{2 \times 11 \times 11.87} = 0.693$, $A = 46.2^\circ$ (keep c unrounded in the cosine rule).

2.5 Find the area of a triangle with sides 6 and 10 and included angle 30° . [2]

3 Exam-style questions

3.1 (Original, 2E cosine family.) In triangle ABC , $a = 7$, $b = 10$ and $C = 66^\circ$. (a) Find c to 3 s.f. (b) Find angle A to 1 d.p. (c) Verify $A + B + C = 180^\circ$. [6]

3.2 (*Original, 2E cos-ratio family.*) In triangle PQR , the sides adjacent to angle Q are $PQ = 11$ cm and $QR = 8$ cm, and $\cos Q = \frac{8}{11}$. (a) Write the cosine rule for $\cos Q$ in terms of PQ , QR and p . (b) Find p to 3 s.f. [4]

3.3 (*Original, 1E 3D family.*) A vertical mast stands at the centre of a horizontal square field of side 20 m. The angle of elevation of the top of the mast from a corner is 38.66° . Find the height of the mast. [4]
