3.3 Mensuration, sectors and solids

Name: _____ Class: _____ Date: _____ **Total: 29 marks**

KEY WORDS sector

Mensuration, sectors and solids

Objective

Use area and volume formulae, circle and sector formulae, and the sphere, pyramid, cone and frustum programme with composite solids (rows G14–G16, G18 with the Extension additions).

1 Key knowledge and worked examples

Areas. Triangle $\frac{1}{2}bh$, parallelogram bh, trapezium $\frac{1}{2}(a+b)h$, circle πr^2 with circumference $2\pi r$; volume $V = Ah$ for constant cross-sections. Sectors (Extension): arc $\frac{\theta}{360} \times 2\pi r$, area $\frac{\theta}{360} \times \pi r^2$.

Solids (Extension). Sphere $V = \frac{4}{3}\pi r^3$, $SA = 4\pi r^2$; cone $V = \frac{1}{3}\pi r^2 h$ with curved surface $\pi r l$; pyramid $V = \frac{1}{3}Ah$; frustums by subtracting the removed cone. A cone $r = 5$, $h = 12$: slant 13, $V = 100\pi$, curved surface 65π.

2 Practice

2.1 Find the area of a trapezium with parallel sides 7 and 11 and height 4 [2]

2.2 A sector has $r = 9$, $\theta = 40°$. Find its arc and area. [3]

2.3 Find the volume and surface area of a sphere of radius 6, in terms of π. [3]

2.4 A cone has $r = 5$ and $h = 12$. Find its slant, volume and curved surface, in terms of π. [4]

2.5 A cylinder has $r = 4$, $h = 10$. Find its volume and total surface area, in terms of π. [3]

3 Exam-style questions

3.1 *(Original, 1E cone-proportion family.)* A frustum is made by removing a cone of radius $3r$ and height $4r$ from a similar cone of radius $5r$ and height $\frac{20}{3}r$. (a) Show that the removed volume is $12\pi r^3$. (b) Show that the frustum's volume is $\frac{392}{9}\pi r^3$. (c) The large cone's radius is 15 cm; find the removed cone's height. [5]

3.2 *(Original.)* A silo is a cylinder ($r = 3$ m, $h = 7$ m) topped by a hemisphere. (a) Find the total volume in terms of π. (b) Find the surface area of the curved surfaces (the silo stands on the ground), in terms of π. [5]

3.3 *(Original.)* A sphere fits exactly inside a closed cylinder (radius r). Show that the ratio of their volumes is $2 : 3$, and find the ratio of their surface areas. [4]

Solutions

2.1 $\frac{1}{2}(7+11)\times 4 = 36$.

2.2 Arc $= \frac{40}{360}\times 18\pi = 2\pi$; area $= \frac{40}{360}\times 81\pi = 9\pi$.

2.3 $V = \frac{4}{3}\pi(216) = 288\pi$; $SA = 4\pi(36) = 144\pi$.

2.4 $l = \sqrt{25+144} = 13$; $V = \frac{1}{3}\pi(25)(12) = 100\pi$; curved $= \pi(5)(13) = 65\pi$.

2.5 $V = \pi(16)(10) = 160\pi$; $SA = 2\pi(16) + 2\pi(4)(10) = 32\pi + 80\pi = 112\pi$.

3.1 (a) Removed cone: $\frac{1}{3}\pi(3r)^2(4r) = \frac{1}{3}\pi\times 36r^3 = 12\pi r^3$. ✓ (b) Large cone: $\frac{1}{3}\pi(5r)^2\left(\frac{20}{3}r\right) = \frac{500}{9}\pi r^3$; frustum $= \frac{500}{9}\pi - \frac{108}{9}\pi = \frac{392}{9}\pi r^3$. ✓ (c) Radius 15 = 5r gives $r = 3$; the large height is $\frac{20}{3}\times 3 = 20$ cm, and similar cones have height ratio $\frac{3}{5}$, so the removed height is $\frac{3}{5}\times 20 = 4r = 12$ cm.

3.2 (a) Cylinder $\pi(9)(7) = 63\pi$; hemisphere $\frac{2}{3}\pi(27) = 18\pi$; total 81π m^3. (b) Cylinder wall $2\pi(3)(7) = 42\pi$; hemisphere curved $2\pi(9) = 18\pi$; curved total 60π m^2 (the ground-covered base is excluded).

3.3 Sphere $V = \frac{4}{3}\pi r^3$; cylinder $V = \pi r^2(2r) = 2\pi r^3$: ratio $\frac{4}{3} : 2 = 2 : 3$. ✓ Sphere $SA = 4\pi r^2$; cylinder total $= 2\pi r^2 + 2\pi r(2r) = 6\pi r^2$: ratio $4 : 6 = 2 : 3$.

Local review guidance: the right formula quoted before substitution; exact π forms; frustum parts label which cone each term belongs to. This is authored classroom guidance.