

# 3.2 Circle theorems with proof

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ Total: 14 marks

KEY WORDS proof

## Circle theorems with proof

### Objective

Apply the standard circle theorems with proof-style reasons, and use geometric reasoning to justify results (rows G8–G9, Extension).

### 1 Key knowledge and worked examples

**The theorem bank.** Centre angle =  $2 \times$  circumference angle (same arc); semicircle angle  $90^\circ$ ; same segment, equal angles; cyclic quadrilateral opposite angles  $180^\circ$ ; tangent perpendicular to the radius; tangents from an external point equal; perpendicular from centre bisects the chord; alternate segment theorem.

**The verified chain.** Circumference angle  $35^\circ \rightarrow$  centre angle  $70^\circ$  (same arc)  $\rightarrow$  the angle from the opposite arc is  $\frac{1}{2}(360^\circ - 70^\circ) = 145^\circ$ , and  $35^\circ + 145^\circ = 180^\circ$  (cyclic quadrilateral): every step names its theorem, in full words.

### 2 Practice

2.1 An angle at the circumference is  $41^\circ$ . Write down the angle at the centre on the same arc. [1]

2.2  $PQ$  is a diameter and  $R$  is on the circle. Find  $\angle PRQ$ . [1]

2.3 Two tangents from an external point  $T$  touch the circle at  $A$  and  $B$ . State the relationship between  $TA$  and  $TB$ . [1]

2.4 The perpendicular from the centre  $O$  to a chord  $XY$  meets it at  $M$  with  $XM = 4.3$  cm. Write down  $MY$ . [1]

### 3 Exam-style questions

3.1 (Original, 1E  $35^\circ/100^\circ$  family.)  $A, B, C, D$  lie on a circle centre  $O$  with  $\angle ADB = 35^\circ$  ( $D$  on the major arc  $AB$ ). (a) Write down  $\angle AOB$ . (b)  $C$  lies on the minor arc  $AB$ : write down  $\angle ACB$ , with reasons. [4]

3.2 (Original.)  $TP$  and  $TQ$  are tangents to a circle centre  $O$  at  $P$  and  $Q$ ;  $\angle PTQ = 46^\circ$ . (a) Explain why  $\angle OPT = 90^\circ$ . (b) Find  $\angle POQ$ . (c) Hence find  $\angle PRQ$  for  $R$  on the major arc  $PQ$ . [4]

3.3 (Original, alternate segment theorem.) The tangent at  $A$  to a circle meets a line through  $B$  (also on the circle) at  $T$ , and  $C$  is on the circle with  $\angle TAB = 62^\circ$ . Write down  $\angle ACB$ , with a reason. [2]

## Solutions

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**2.1**  $82^\circ$ .

**2.2**  $90^\circ$  (angle in a semicircle).

**2.3**  $TA = TB$  (tangents from an external point are equal).

**2.4** 4.3 cm (the perpendicular from the centre bisects the chord).

**3.1** (a)  $\angle AOB = 2 \times 35^\circ = 70^\circ$  (centre angle is twice the circumference angle on the same arc). (b)  $\angle ACB$  subtends the same chord from the OTHER arc, so it is half the reflex angle at the centre:  $\frac{1}{2}(360^\circ - 70^\circ) = 145^\circ$  – the two circumference angles are opposite angles of a cyclic quadrilateral summing to  $180^\circ$ . ✓

**3.2** (a) The tangent at any point is perpendicular to the radius at that point. (b) The quadrilateral  $OPTQ$  has angles  $90^\circ + 90^\circ + 46^\circ + \angle POQ = 360^\circ$ , so  $\angle POQ = 134^\circ$ . (c)  $\angle PRQ = \frac{1}{2} \times 134^\circ = 67^\circ$  (centre is double the circumference on the same arc,  $R$  on the major arc).

**3.3**  $\angle ACB = 62^\circ$  – the alternate segment theorem: the angle between the tangent and the chord equals the angle in the alternate segment.

Local review guidance: the theorem named in full words at every step; exact angles; cyclic-quadrilateral and reflex-arc cases justified. This is authored classroom guidance.