

2.2 Functions and proof

Name: _____ Class: _____ Date: _____

Total: 19 marks

KEY WORDS domain • proof

Functions and proof

Objective

Use function notation with domain and range, build composites and inverses, and construct formal algebraic proofs (rows A8–A9 with the Extension additions).

1 Key knowledge and worked examples

Functions. $f(x)$ maps each input to one output; domain is the input set, range the output set. Composite $fg(x) = f(g(x))$ – apply g first. Inverse by swap-and-rearrange; the 1E specimen has $f^{-1}(x) = \frac{x}{3}$ from $f(x) = 3x$.

Proof. State the algebra, simplify, compare with the claim: $(2n+1)^2 - (2n-1)^2 = 8n$ – a multiple of 8 for every integer n . The 2E specimen proves the $n^2 + n + 9$ family. Examples never prove.

2 Practice

2.1 For $f(x) = 3x - 5$, find $f(4)$ [1]

2.2 For $f(x) = 3x - 5$ and $g(x) = x^2$, find $fg(2)$ [2]

2.3 Find $f^{-1}(x)$ for $f(x) = 3x$ [2]

2.4 State the domain and range of $f(x) = \sqrt{x-2}$ [2]

3 Exam-style questions

3.1 (*Original, 2E composite family.*) $g(x) = x - 5$ and $f(x) = x^2 + 1$. (a) Find $fg(x)$. (b) Solve $fg(x) = 5$. [4]

3.2 (*Original, 1E inverse family.*) $f(x) = \frac{x}{3} + 2$. (a) Find $f^{-1}(x)$. (b) Verify that $f^{-1}(14) = 36$. [4]

3.3 (*Original, 2E proof family.*) Prove that the difference of the squares of any two consecutive odd numbers is always a multiple of 8. [4]

Solutions

2.1 7.

2.2 $g(2) = 4$, then $f(4) = 7$.

2.3 $f^{-1}(x) = \frac{x}{3}$.

2.4 Domain $x \geq 2$; range $f(x) \geq 0$.

3.1 (a) $fg(x) = (x - 5)^2 + 1$. (b) $(x - 5)^2 = 4$, so $x = 3$ or $x = 7$.

3.2 (a) $y = \frac{x}{3} + 2 \Rightarrow x = 3(y - 2)$, so $f^{-1}(x) = 3x - 6$. (b) $f^{-1}(14) = 42 - 6 = 36$. ✓

3.3 Consecutive odd numbers: $2n + 1$ and $2n - 1$. Their squares differ by $(2n + 1)^2 - (2n - 1)^2 = (4n^2 + 4n + 1) - (4n^2 - 4n + 1) = 8n$, a multiple of 8 for every integer n .

Local review guidance: composite order – g first inside f ; inverses with domains; proofs end with the conclusion in words. This is authored classroom guidance.