

Exercise walk-through

3.5

Transformations, vectors and matrices

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1. Unit 3.3 at a glance

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Core G1-G26: angle facts through constructions, mensuration, similarity, trigonometry, transformations, vectors and matrices. Extension adds the full circle-theorem programme with proof, congruence conditions, bearings, sphere/pyramid/cone/frustum formulae, k - k^2 - k^3 similarity, sectors, 3D Pythagoras and trigonometry, the sine/cosine rules with half-angle, combined transformations with fractional and negative factors, vector geometry, and the 2×2 matrix programme.

Must-know (no formulae sheet): $2\pi r$ and πr^2 ; sector fractions of 360; sphere $\frac{4}{3}\pi r^3$ and $4\pi r^2$; $a^2 = b^2 + c^2 - 2bc \cos A$; sine rule; Area = half $ab \sin C$.

7. Transformations, vectors, matrices

Transformations, vectors, matrices

Transformations with combined and fractional/negative scale factors (Extension): an inverted, shrunk image on the opposite side of the centre. Vectors: sums, differences, scalar multiples, simple geometry - the 2E specimen proves CB parallel MN via a half ratio.

2x2 matrices (Extension): multiplications, the identity I , unit-square transformations - rotation 90: $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$; reflection in $y = -x$: $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ (the 1E specimen's matrix); enlargement k : $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$. Combinations multiply in order; solve $Mx = b$ as simultaneous equations (the 1E pairs $(6, -1/2)$ and $(-2, -5/2)$).

Check yourself

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Angle facts and polygon sums; quadrilaterals and symmetry; congruence conditions; similarity ratios in every direction; the circle-theorem bank with proof reasons.

Solid properties; plans and elevations; bearings; constructions and loci; area, volume, sector, sphere, cone and frustum formulae; 2D and 3D trigonometry with the sine and cosine rules; transformations including negative factors; vector geometry; matrix multiplication, identity, unit-square transformations and combinations.

Exercises 2.1·2.3

2.1 Rotate $(-2, 4)$ by 90° anticlockwise about O [2]

2.2 Write the matrix for a reflection in the x -axis. [2]

2.3 Multiply $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ [2]

Exercises 2.1·2.3

2.1 Rotate $(-2, 4)$ by 90° anticlockwise about O [2]

2.1 $(x, y) \rightarrow (-y, x)$: $(-4, -2)$.

2.2 Write the matrix for a reflection in the x -axis. [2]

2.2 $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

2.3 Multiply $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ [2]

2.3 $\begin{pmatrix} 10 \\ 5 \end{pmatrix}$.

Exercises 2.4·3.1

2.4 $\overrightarrow{AB} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$. Find $\overrightarrow{AC}$. [2]

2.5 Solve the matrix equation $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$ [3]

3.1 (*Original, 1E matrix family.*) The transformation T is represented by $\mathbf{M} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.
(a) Find the image of the unit square's vertices under T . (b) Describe T fully. (c) Verify that $\mathbf{M}^2 = I$. [4]

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2.4 $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$.

2.5 Solve the matrix equation $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$ [3]

2.5 $3x + y = 10$, $x + 2y = 5$: $y = 10 - 3x$ gives $x + 20 - 6x = 5$, so $x = 3$, $y = 1$.

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 (a) Find the image of the unit square's vertices under T . (b) Describe T fully. (c) Verify that $\mathbf{M}^2 = I$. [4]

3.1 (a) $(0, 0) \rightarrow (0, 0)$; $(1, 0) \rightarrow (0, -1)$; $(0, 1) \rightarrow (-1, 0)$; $(1, 1) \rightarrow (-1, -1)$. (b) Reflection in the line $y = -x$. (c) $\mathbf{M}^2 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$. ✓

Exercises 3.2·3.3

3.2 (*Original, 1E matrix-solve family.*) Solve $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$, giving fractions where exact. [3]

3.3 (*Original, 2E vector-proof family.*) In triangle OCB , M is the midpoint of OB and N is the midpoint of CB . With $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OB} = \mathbf{b}$, show that $\overrightarrow{MN} = \frac{1}{2}\mathbf{c}$ and state what this proves about MN and OC . [4]

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3.2 $2x + 3y = 0$ and $x + 4y = 5$: from the first $x = -\frac{3}{2}y$; then $-\frac{3}{2}y + 4y = 5$, so $\frac{5}{2}y = 5$, $y = 2$, $x = -3$: $(x, y) = (-3, 2)$.

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3.3 $\overrightarrow{OM} = \frac{1}{2}\mathbf{b}$; $\overrightarrow{ON} = \frac{1}{2}(\mathbf{b} + \mathbf{c})$. So $\overrightarrow{MN} = \overrightarrow{ON} - \overrightarrow{OM} = \frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{c} - \frac{1}{2}\mathbf{b} = \frac{1}{2}\mathbf{c} = \frac{1}{2}\overrightarrow{OC}$: MN is parallel to OC and half its length.

Local review guidance: matrices multiplied in the order the transformations apply; exact coordinates and fractions; vector proofs end with the parallel-and-length statement. This is authored classroom guidance.