

Exercise walk-through

3.4

Trigonometry in 2D and 3D

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1. Unit 3.3 at a glance

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Core G1-G26: angle facts through constructions, mensuration, similarity, trigonometry, transformations, vectors and matrices. Extension adds the full circle-theorem programme with proof, congruence conditions, bearings, sphere/pyramid/cone/frustum formulae, k - k^2 - k^3 similarity, sectors, 3D Pythagoras and trigonometry, the sine/cosine rules with half-angle, combined transformations with fractional and negative factors, vector geometry, and the 2×2 matrix programme.

Must-know (no formulae sheet): $2\pi r$ and πr^2 ; sector fractions of 360; sphere $\frac{4}{3}\pi r^3$ and $4\pi r^2$; $a^2 = b^2 + c^2 - 2bc \cos A$; sine rule; Area = half $ab \sin C$.

6. Trigonometry 2D and 3D

Trigonometry 2D and 3D

Pythagoras and SOH CAH TOA in right triangles; 3D problems (Extension) extract the right triangle first - the 1E specimen ends at 62.7 degrees.

Sine and cosine rules with Area = half $a b \sin C$ (Extension): the 2E cosine rule gives 102.6 degrees, and $\cos x = 8/11$ gives 43.3 degrees. Worked chain: $a = 9$, $b = 11$, $C = 72$ gives $c = 11.9$, then $A = 46.3$ with the angle sum checking 180.

Check yourself

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Angle facts and polygon sums; quadrilaterals and symmetry; congruence conditions; similarity ratios in every direction; the circle-theorem bank with proof reasons.

Solid properties; plans and elevations; bearings; constructions and loci; area, volume, sector, sphere, cone and frustum formulae; 2D and 3D trigonometry with the sine and cosine rules; transformations including negative factors; vector geometry; matrix multiplication, identity, unit-square transformations and combinations.

Exercises 2.1·2.3

2.1 A right triangle has legs 5 and 12. Find the hypotenuse. [2]

2.2 Find θ where the opposite side is 7 and the hypotenuse is 9, to 1 d.p. [2]

2.3 A triangle has $a = 8$, $b = 11$, $C = 60^\circ$. Find c . [3]

Exercises 2.1·2.3

2.1 A right triangle has legs 5 and 12. Find the hypotenuse. [2]

2.1 $\sqrt{25 + 144} = 13$.

2.2 Find θ where the opposite side is 7 and the hypotenuse is 9, to 1 d.p. [2]

2.2 $\sin \theta = \frac{7}{9}$: $\theta = 51.1^\circ$.

2.3 A triangle has $a = 8$, $b = 11$, $C = 60^\circ$. Find c . [3]

2.3 $c^2 = 64 + 121 - 176 \cos 60^\circ = 185 - 88 = 97$: $c = 9.85$.

Exercises 2.4·3.1

2.4 $c^2 = 81 + 121 - 198 \cos 72^\circ = 202 - 61.2 = 140.8$: $c = 11.9$; smallest angle opposite the shortest side: $\cos A = \frac{121+140.8-81}{2 \times 11 \times 11.87} = 0.693$, $A = 46.2^\circ$ (keep c unrounded in the cosine rule).

2.5 Find the area of a triangle with sides 6 and 10 and included angle 30° . [2]

3.1 (*Original, 2E cosine family.*) In triangle ABC , $a = 7$, $b = 10$ and $C = 66^\circ$. (a) Find c to 3 s.f. (b) Find angle A to 1 d.p. (c) Verify $A + B + C = 180^\circ$. [6]

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2.5 Find the area of a triangle with sides 6 and 10 and included angle 30° . [2]

2.5 $\frac{1}{2} \times 6 \times 10 \times \sin 30^\circ = 15$.

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3.1 (a) $c^2 = 49 + 100 - 140 \cos 66^\circ = 92.1$, so $c = 9.59$. (b) $\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{100 + 92.1 - 49}{2 \times 10 \times 9.595} = 0.745$, so $A = 41.8^\circ$. (c) $B = 180 - 66 - 41.8 = 72.2^\circ$: $41.8 + 72.2 + 66 = 180$. ✓

Exercises 3.2·3.3

3.2 (*Original, 2E cos-ratio family.*) In triangle PQR , the sides adjacent to angle Q are $PQ = 11$ cm and $QR = 8$ cm, and $\cos Q = \frac{8}{11}$. (a) Write the cosine rule for $\cos Q$ in terms of PQ , QR and p . (b) Find p to 3 s.f. [4]

3.3 (*Original, 1E 3D family.*) A vertical mast stands at the centre of a horizontal square field of side 20 m. The angle of elevation of the top of the mast from a corner is 38.66° . Find the height of the mast. [4]

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3.2 (a) $\cos Q = \frac{PQ^2 + QR^2 - p^2}{2(PQ)(QR)}$. (b) $\frac{185 - p^2}{176} = \frac{8}{11} = \frac{128}{176}$, so $p^2 = 57$ and $p = 7.55$ cm. Check: $185 - 57 = 128$ and $\frac{128}{176} = \frac{8}{11}$. ✓

3.3 (*Original, 1E 3D family.*) A vertical mast stands at the centre of a horizontal square field of side 20 m. The angle of elevation of the top of the mast from a corner is 38.66° . Find the height of the mast. [4]

3.3 Half-diagonal of the field: $\frac{1}{2} \times 20\sqrt{2} = 14.14$ m; $\tan 38.66^\circ = \frac{h}{14.14}$, so $h = 14.14 \times 0.800 = 11.3$ m.

Local review guidance: the correct rule for the given parts; 3 s.f. accuracy with the angle to 1 d.p.; 3D problems name the extracted triangle. This is authored classroom guidance.