

Exercise walk-through

3.3

Mensuration, sectors and solids

1. Unit 3.3 at a glance

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Core G1-G26: angle facts through constructions, mensuration, similarity, trigonometry, transformations, vectors and matrices. Extension adds the full circle-theorem programme with proof, congruence conditions, bearings, sphere/pyramid/cone/frustum formulae, k - k^2 - k^3 similarity, sectors, 3D Pythagoras and trigonometry, the sine/cosine rules with half-angle, combined transformations with fractional and negative factors, vector geometry, and the 2×2 matrix programme.

Must-know (no formulae sheet): $2\pi r$ and πr^2 ; sector fractions of 360; sphere $\frac{4}{3}\pi r^3$ and $4\pi r^2$; $a^2 = b^2 + c^2 - 2bc \cos A$; sine rule; Area = half $ab \sin C$.

3. Congruence, similarity, circles

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Congruence conditions (Extension): SSS, SAS, ASA, RHS. Similarity: lengths k , areas k^2 , volumes k^3 - the 2E specimen's similar solids give 864 cm cubed.

Circle theorems (Extension): centre double circumference; semicircle 90; same segment equal; cyclic quadrilateral opposite angles 180; tangent perpendicular radius; equal tangents; centre-perpendicular bisects chord; alternate segment. The verified chain: 35 degrees at the circumference gives 70 at the centre, and the opposite-arc angle is half of 290 = 145, with $35 + 145 = 180$ (cyclic quadrilateral) - every step named.

5. Mensuration and similarity

Mensuration and similarity

Areas: half-b-h, b-h, half(a+b)h; volume $V = Ah$; circle $2\pi r$ and πr^2 .

Sectors (Extension): $\frac{\theta}{360}$ of the circle. The 1E cone proportion finds $p = 15$.

Solids (Extension): sphere, pyramid, cone with frustums by subtraction - a cone $r = 5$, $h = 12$ has slant 13, volume 100π and curved surface 65π . Similar ratios: lengths k , areas k^2 , volumes k^3 .

Exercises 2.1·2.3

2.1 Find the area of a trapezium with parallel sides 7 and 11 and height 4 [2]

2.2 A sector has $r = 9$, $\theta = 40^\circ$. Find its arc and area. [3]

2.3 Find the volume and surface area of a sphere of radius 6, in terms of π . [3]

Exercises 2.1·2.3

2.1 Find the area of a trapezium with parallel sides 7 and 11 and height 4 [2]

$$\mathbf{2.1} \quad \frac{1}{2}(7 + 11) \times 4 = 36.$$

2.2 A sector has $r = 9$, $\theta = 40^\circ$. Find its arc and area. [3]

$$\mathbf{2.2} \quad \text{Arc} = \frac{40}{360} \times 18\pi = 2\pi; \text{ area} = \frac{40}{360} \times 81\pi = 9\pi.$$

2.3 Find the volume and surface area of a sphere of radius 6, in terms of π . [3]

$$\mathbf{2.3} \quad V = \frac{4}{3}\pi(216) = 288\pi; SA = 4\pi(36) = 144\pi.$$

Exercises 2.4·3.1

2.4 A cone has $r = 5$ and $h = 12$. Find its slant, volume and curved surface, in terms of π . [4]

2.5 A cylinder has $r = 4$, $h = 10$. Find its volume and total surface area, in terms of π . [3]

3.1 (*Original, 1E cone-proportion family.*) A frustum is made by removing a cone of radius $3r$ and height $4r$ from a similar cone of radius $5r$ and height $\frac{20}{3}r$. (a) Show that the removed volume is $12\pi r^3$. (b) Show that the frustum's volume is $\frac{392}{9}\pi r^3$. (c) The large cone's radius is 15 cm; find the removed cone's height. [5]

Exercises 2.4.3.1

2.4 A cone has $r = 5$ and $h = 12$. Find its slant, volume and curved surface, in terms of π . [4]

$$\mathbf{2.4} \quad l = \sqrt{25 + 144} = 13; V = \frac{1}{3}\pi(25)(12) = 100\pi; \text{ curved} = \pi(5)(13) = 65\pi.$$

2.5 A cylinder has $r = 4$, $h = 10$. Find its volume and total surface area, in terms of π . [3]

$$\mathbf{2.5} \quad V = \pi(16)(10) = 160\pi; SA = 2\pi(16) + 2\pi(4)(10) = 32\pi + 80\pi = 112\pi.$$

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3.1 (a) Removed cone: $\frac{1}{3}\pi(3r)^2(4r) = \frac{1}{3}\pi \times 36r^3 = 12\pi r^3$. ✓ (b) Large cone: $\frac{1}{3}\pi(5r)^2\left(\frac{20}{3}r\right) = \frac{500}{9}\pi r^3$; frustum = $\frac{500}{9}\pi - \frac{108}{9}\pi = \frac{392}{9}\pi r^3$. ✓ (c) Radius 15 = $5r$ gives $r = 3$; the large height is $\frac{20}{3} \times 3 = 20$ cm, and similar cones have height ratio $\frac{3}{5}$, so the removed height is $\frac{3}{5} \times 20 = 4r = 12$ cm.

Exercises 3.2·3.3

3.2 (*Original.*) A silo is a cylinder ($r = 3$ m, $h = 7$ m) topped by a hemisphere. (a) Find the total volume in terms of π . (b) Find the surface area of the curved surfaces (the silo stands on the ground), in terms of π . [5]

3.3 (*Original.*) A sphere fits exactly inside a closed cylinder (radius r). Show that the ratio of their volumes is $2 : 3$, and find the ratio of their surface areas. [4]

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3.2 (a) Cylinder $\pi(9)(7) = 63\pi$; hemisphere $\frac{2}{3}\pi(27) = 18\pi$; total 81π m³. (b) Cylinder wall $2\pi(3)(7) = 42\pi$; hemisphere curved $2\pi(9) = 18\pi$; curved total 60π m² (the ground-covered base is excluded).

3.3 (*Original.*) A sphere fits exactly inside a closed cylinder (radius r). Show that the ratio of their volumes is $2 : 3$, and find the ratio of their surface areas. [4]

3.3 Sphere $V = \frac{4}{3}\pi r^3$; cylinder $V = \pi r^2(2r) = 2\pi r^3$: ratio $\frac{4}{3} : 2 = 2 : 3$. ✓ Sphere $SA = 4\pi r^2$; cylinder total $= 2\pi r^2 + 2\pi r(2r) = 6\pi r^2$: ratio $4 : 6 = 2 : 3$.

Local review guidance: the right formula quoted before substitution; exact π forms; frustum parts label which cone each term belongs to. This is authored classroom guidance.