

Exercise walk-through

3.2

Circle theorems with proof

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1. Unit 3.3 at a glance

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Core G1-G26: angle facts through constructions, mensuration, similarity, trigonometry, transformations, vectors and matrices. Extension adds the full circle-theorem programme with proof, congruence conditions, bearings, sphere/pyramid/cone/frustum formulae, k - k^2 - k^3 similarity, sectors, 3D Pythagoras and trigonometry, the sine/cosine rules with half-angle, combined transformations with fractional and negative factors, vector geometry, and the 2×2 matrix programme.

Must-know (no formulae sheet): $2\pi r$ and πr^2 ; sector fractions of 360; sphere $\frac{4}{3}\pi r^3$ and $4\pi r^2$; $a^2 = b^2 + c^2 - 2bc \cos A$; sine rule; Area = half $ab \sin C$.

3. Congruence, similarity, circles

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Congruence conditions (Extension): SSS, SAS, ASA, RHS. Similarity: lengths k , areas k^2 , volumes k^3 - the 2E specimen's similar solids give 864 cm cubed.

Circle theorems (Extension): centre double circumference; semicircle 90; same segment equal; cyclic quadrilateral opposite angles 180; tangent perpendicular radius; equal tangents; centre-perpendicular bisects chord; alternate segment. The verified chain: 35 degrees at the circumference gives 70 at the centre, and the opposite-arc angle is half of 290 = 145, with $35 + 145 = 180$ (cyclic quadrilateral) - every step named.

Check yourself

Check yourself

Angle facts and polygon sums; quadrilaterals and symmetry; congruence conditions; similarity ratios in every direction; the circle-theorem bank with proof reasons.

Solid properties; plans and elevations; bearings; constructions and loci; area, volume, sector, sphere, cone and frustum formulae; 2D and 3D trigonometry with the sine and cosine rules; transformations including negative factors; vector geometry; matrix multiplication, identity, unit-square transformations and combinations.

Exercises 2.1·2.3

2.1 An angle at the circumference is 41° . Write down the angle at the centre on the same arc. [1]

2.2 PQ is a diameter and R is on the circle. Find $\angle PRQ$. [1]

2.3 Two tangents from an external point T touch the circle at A and B . State the relationship between TA and TB . [1]

Exercises 2.1·2.3

2.1 An angle at the circumference is 41° . Write down the angle at the centre on the same arc. [1]

2.1 82° .

2.2 PQ is a diameter and R is on the circle. Find $\angle PRQ$. [1]

2.2 90° (angle in a semicircle).

2.3 Two tangents from an external point T touch the circle at A and B . State the relationship between TA and TB . [1]

2.3 $TA = TB$ (tangents from an external point are equal).

Exercises 2.4·3.1

2.4 The perpendicular from the centre O to a chord XY meets it at M with $XM = 4.3$ cm. Write down MY . [1]

3.1 (*Original, 1E $35^\circ/100^\circ$ family.*) A, B, C, D lie on a circle centre O with $\angle ADB = 35^\circ$ (D on the major arc AB). (a) Write down $\angle AOB$. (b) C lies on the minor arc AB : write down $\angle ACB$, with reasons. [4]

Exercises 2.4·3.1

2.4 The perpendicular from the centre O to a chord XY meets it at M with $XM = 4.3$ cm. Write down MY . [1]

2.4 4.3 cm (the perpendicular from the centre bisects the chord).

3.1 (*Original, 1E $35^\circ/100^\circ$ family.*) A, B, C, D lie on a circle centre O with $\angle ADB = 35^\circ$ (D on the major arc AB). (a) Write down $\angle AOB$. (b) C lies on the minor arc AB : write down $\angle ACB$, with reasons. [4]

3.1 (a) $\angle AOB = 2 \times 35^\circ = 70^\circ$ (centre angle is twice the circumference angle on the same arc). (b) $\angle ACB$ subtends the same chord from the OTHER arc, so it is half the reflex angle at the centre: $\frac{1}{2}(360^\circ - 70^\circ) = 145^\circ$ – the two circumference angles are opposite angles of a cyclic quadrilateral summing to 180° . ✓

Exercises 3.2·3.3

3.2 (*Original.*) TP and TQ are tangents to a circle centre O at P and Q ; $\angle PTQ = 46^\circ$.

(a) Explain why $\angle OPT = 90^\circ$. (b) Find $\angle POQ$. (c) Hence find $\angle PRQ$ for R on the major arc PQ . [4]

3.3 (*Original, alternate segment theorem.*) The tangent at A to a circle meets a line through B (also on the circle) at T , and C is on the circle with $\angle TAB = 62^\circ$. Write down $\angle ACB$, with a reason. [2]

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(a) Explain why $\angle OPT = 90^\circ$. (b) Find $\angle POQ$. (c) Hence find $\angle PRQ$ for R on the major arc PQ . [4]

3.2 (a) The tangent at any point is perpendicular to the radius at that point. (b) The quadrilateral $OPTQ$ has angles $90^\circ + 90^\circ + 46^\circ + \angle POQ = 360^\circ$, so $\angle POQ = 134^\circ$. (c) $\angle PRQ = \frac{1}{2} \times 134^\circ = 67^\circ$ (centre is double the circumference on the same arc, R on the major arc).

3.3 (*Original, alternate segment theorem.*) The tangent at A to a circle meets a line through B (also on the circle) at T , and C is on the circle with $\angle TAB = 62^\circ$. Write down $\angle ACB$, with a reason. [2]

3.3 $\angle ACB = 62^\circ$ – the alternate segment theorem: the angle between the tangent and the chord equals the angle in the alternate segment.

Local review guidance: the theorem named in full words at every step; exact angles; cyclic-quadrilateral and reflex-arc cases justified. This is authored classroom guidance.