

Exercise walk-through

3.1

Angles, polygons, congruence and similarity

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1. Unit 3.3 at a glance

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Core G1-G26: angle facts through constructions, mensuration, similarity, trigonometry, transformations, vectors and matrices. Extension adds the full circle-theorem programme with proof, congruence conditions, bearings, sphere/pyramid/cone/frustum formulae, k - k^2 - k^3 similarity, sectors, 3D Pythagoras and trigonometry, the sine/cosine rules with half-angle, combined transformations with fractional and negative factors, vector geometry, and the 2×2 matrix programme.

Must-know (no formulae sheet): $2\pi r$ and πr^2 ; sector fractions of 360; sphere $\frac{4}{3}\pi r^3$ and $4\pi r^2$; $a^2 = b^2 + c^2 - 2bc \cos A$; sine rule; Area = half $ab \sin C$.

2. Angles and polygons

Angles and polygons

Point 360, line 180, vertically opposite equal; alternate, corresponding, interior angles between parallels (never 'Z angles'); triangle sum 180 with the isosceles, equilateral and scalene families.

Polygon sums: interior $(n-2) \times 180$, exterior $360/n$, centre angle $360/n$ for regular polygons. Quadrilateral properties and symmetry counts: a regular hexagon has 6 lines and order 6.

3. Congruence, similarity, circles

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Congruence conditions (Extension): SSS, SAS, ASA, RHS. Similarity: lengths k , areas k^2 , volumes k^3 - the 2E specimen's similar solids give 864 cm cubed.

Circle theorems (Extension): centre double circumference; semicircle 90; same segment equal; cyclic quadrilateral opposite angles 180; tangent perpendicular radius; equal tangents; centre-perpendicular bisects chord; alternate segment. The verified chain: 35 degrees at the circumference gives 70 at the centre, and the opposite-arc angle is half of 290 = 145, with $35 + 145 = 180$ (cyclic quadrilateral) - every step named.

Exercises 2.1·2.3

2.1 A regular polygon has interior angle 150° . Find n and the angle at the centre. [3]

2.2 Two angles of a triangle are 47° and 71° . Find the third and the exterior angle at that vertex. [2]

2.3 Write down the congruence condition for two triangles with two sides and the included angle equal. [1]

Exercises 2.1·2.3

2.1 A regular polygon has interior angle 150° . Find n and the angle at the centre. [3]

2.1 Exterior $= 30^\circ$, so $n = 12$; centre angle $= 30^\circ$.

2.2 Two angles of a triangle are 47° and 71° . Find the third and the exterior angle at that vertex. [2]

2.2 Third angle $= 62^\circ$; exterior $= 47 + 71 = 118^\circ$.

2.3 Write down the congruence condition for two triangles with two sides and the included angle equal. [1]

2.3 SAS.

Exercises 2.4·3.1

2.4 Similar jugs have heights 8 cm and 12 cm. The smaller holds 144 cm^3 . Find the larger's volume. [3]

2.5 Similar photographs have areas 18 cm^2 and 50 cm^2 . Find the ratio of their widths. [3]

3.1 (*Original, 2E SAS family.*) Triangles ABC and PQR have $AB = PQ = 7 \text{ cm}$, $BC = QR = 9 \text{ cm}$ and $\angle ABC = \angle PQR = 54^\circ$. (a) Prove the triangles are congruent. (b) Write down the length AC . [3]

Exercises 2.4·3.1

2.4 Similar jugs have heights 8 cm and 12 cm. The smaller holds 144 cm^3 . Find the larger's volume. [3]

2.4 $k = \frac{12}{8} = \frac{3}{2}$; volumes scale by $\frac{27}{8}$: $144 \times \frac{27}{8} = 486 \text{ cm}^3$.

2.5 Similar photographs have areas 18 cm^2 and 50 cm^2 . Find the ratio of their widths. [3]

2.5 $\sqrt{18} : \sqrt{50} = 3\sqrt{2} : 5\sqrt{2} = 3 : 5$.

3.1 (*Original, 2E SAS family.*) Triangles ABC and PQR have $AB = PQ = 7 \text{ cm}$, $BC = QR = 9 \text{ cm}$ and $\angle ABC = \angle PQR = 54^\circ$. (a) Prove the triangles are congruent. (b) Write down the length AC . [3]

3.1 (a) Two pairs of sides and the included angle are equal: SAS, so the triangles are congruent. (b) $AC = PR$ (corresponding sides) – but PR is given as 9 cm only if the correspondence maps $BC \rightarrow QR$; from SAS with $AB \leftrightarrow PQ$ and $BC \leftrightarrow QR$, the third sides correspond: $AC = PR$. With the data given, the cosine rule gives $AC^2 = 49 + 81 - 2(7)(9) \cos 54^\circ = 130 - 126 \times 0.588 = 130 - 74.1 = 55.9$, so $AC = 7.5 \text{ cm}$ – and PR equals the same value by the same working.

Exercises 3.2·3.3

3.2 (*Original, 2E similar-solids family.*) Two similar statues have heights in the ratio $3 : 5$; the smaller displaces 216 cm^3 of water. (a) Find the volume of the larger. (b) The larger's base area is 150 cm^2 ; find the smaller's. [4]

3.3 (*Original.*) Two lines are cut by a transversal: the two interior angles on one side of the transversal measure 63° and 117° . Explain, with reasons, why the two lines are parallel. [3]

Exercises 3.2-3.3

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3.2 (a) $k = \frac{5}{3}$; volume ratio $\frac{125}{27}$: $216 \times \frac{125}{27} = 1000 \text{ cm}^3$. (b) Area ratio $\frac{25}{9}$: smaller base $= 150 \times \frac{9}{25} = 54 \text{ cm}^2$.

3.3 (*Original.*) Two lines are cut by a transversal: the two interior angles on one side of the transversal measure 63° and 117° . Explain, with reasons, why the two lines are parallel. [3]

3.3 Co-interior angles summing to 180° ($63^\circ + 117^\circ$) mean the two lines are parallel – the interior-angle property between parallels.

Local review guidance: the named fact before the number; exact values; congruence names the condition and the correspondence. This is authored classroom guidance.