

Exercise walk-through

2.6

Sequences

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1. Unit 3.2 at a glance

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Core A1-A26: notation through graphs, equations, inequalities and sequences.

Extension additions: subject-twice rearrangement, triple expansions, all quadratic factorising, fractional powers, quadratic fractions, formal proof, the full functions toolkit, line equations, exponential and trig graphs, differentiation with tangents and stationary points, completing the square, variation, all simultaneity, quadratic inequalities, quadratic sequences.

Must-know: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $a(x + b)^2 + c$ reads the turning point; $y - y_1 = m(x - x_1)$; $\frac{d}{dx}(kx^n) = knx^{(n-1)}$; the tangent's gradient IS the gradient at the point.

7. Curve families and turning points

Curve families and turning points

Sketch and interpret linear, quadratic, cubic and reciprocal graphs; Extension adds exponential $y = k^x$ and the trigonometric graphs in degrees (specification lines, not sampled by the specimens).

Roots, intercepts and turning points read from sketches; turning points DEDUCED by completing the square: $x^2 - 6x + 5 = (x - 3)^2 - 4$ has its minimum at (3, -4). Gradients and areas under graphs estimated by tangents and trapezia.

10. Sequences

Sequences

Term-to-term and position-to-term rules; triangular, square, cube sequences.

Quadratic sequences (Extension): constant second difference, $a = \text{half of it}$.

5, 9, 17, 29, 45: second difference 4 so $a = 2$; subtract $2n^2$ and the residual $-2n + 5$ gives $2n^2 - 2n + 5$. Check on two positions before printing.

Exercises 2.1·2.3

2.1 Write the next two terms of 5, 9, 17, 29, 45, ... [2]

2.2 Find the n th term of 4, 7, 12, 19, 28, ... [3]

2.3 Find the n th term of 6, 11, 18, 27, 38, ... [3]

Exercises 2.1·2.3

2.1 Write the next two terms of 5, 9, 17, 29, 45, ... [2]

2.1 Differences are 4, 8, 12, 16, next differences 20, 24: terms 65, 89.

2.2 Find the n th term of 4, 7, 12, 19, 28, ... [3]

2.2 Differences 3, 5, 7, 9; second difference 2, so $a = 1$: n^2 leaves 3, 3, 3, 3 – n th term $n^2 + 3$.

2.3 Find the n th term of 6, 11, 18, 27, 38, ... [3]

2.3 Differences 5, 7, 9, 11; second difference 2, so $a = 1$. Subtracting n^2 (i.e. 1, 4, 9, 16) leaves 5, 7, 9, 11 = $2n + 3$: n th term $n^2 + 2n + 3$.

Exercises 2.4·3.2

2.4 Which square number follows 144?

[1]

3.1 (*Original, quadratic-sequence family.*) A sequence has n th term $3n^2 - 2n$. (a) Write down the first four terms. (b) Which term of the sequence equals 133? [4]

3.2 (*Original.*) The first differences of a sequence are 5, 9, 13, 17, ... (a) Find the next two differences. (b) Given the first term is 3, write the first four terms. (c) Find the n th term. [5]

Exercises 2.4·3.2

2.4 Which square number follows 144?

[1]

2.4 169.

3.1 (*Original, quadratic-sequence family.*) A sequence has n th term $3n^2 - 2n$. (a) Write down the first four terms. (b) Which term of the sequence equals 133? [4]

3.1 (a) $n = 1$: 1; $n = 2$: 8; $n = 3$: 21; $n = 4$: 40. (b) $3n^2 - 2n = 133$ gives $3n^2 - 2n - 133 = 0$, and $(3n + 19)(n - 7) = 0$, so $n = 7$ (reject $-\frac{19}{3}$). Check: $3(49) - 14 = 133$. ✓

3.2 (*Original.*) The first differences of a sequence are 5, 9, 13, 17, ... (a) Find the next two differences. (b) Given the first term is 3, write the first four terms. (c) Find the n th term. [5]

3.2 (a) The differences themselves increase by 4: next differences 21 and 25. (b) Terms: 3, $3 + 5 = 8$, $8 + 9 = 17$, $17 + 13 = 30$. (c) Second difference of the sequence is 4, so $a = 2$; subtracting $2n^2$ leaves 1, 0, -1, -2 = $-n + 2$: n th term $2n^2 - n + 2$ (check $n = 4$: $32 - 4 + 2 = 30$).

Exercise 3.3

3.3 (*Original, specification line not sampled: variation.*) y varies directly as the square of x ; $y = 48$ when $x = 4$. (a) Find the equation. (b) Find y when $x = 7$. [4]

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3.3 (a) $y = kx^2$: $48 = 16k$ gives $k = 3$, so $y = 3x^2$. (b) $x = 7$: $y = 3 \times 49 = 147$.

Local review guidance: second differences before guessing a ; n th terms checked on two positions; variation parts state the model form. This is authored classroom guidance.