

Exercise walk-through

2.5

Equations, simultaneity and inequalities

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1. Unit 3.2 at a glance

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Core A1-A26: notation through graphs, equations, inequalities and sequences.

Extension additions: subject-twice rearrangement, triple expansions, all quadratic factorising, fractional powers, quadratic fractions, formal proof, the full functions toolkit, line equations, exponential and trig graphs, differentiation with tangents and stationary points, completing the square, variation, all simultaneity, quadratic inequalities, quadratic sequences.

Must-know: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $a(x + b)^2 + c$ reads the turning point; $y - y_1 = m(x - x_1)$; $\frac{d}{dx}(kx^n) = knx^{(n-1)}$; the tangent's gradient IS the gradient at the point.

9. Equations and inequalities

Equations and inequalities

Quadratics by factorising, completing the square, or the formula: $x^2 + 6x + 2 = 0$ gives $x = -3 \pm \sqrt{7}$ (the 1E specimen family). Pairs: two linear by elimination; linear with quadratic by substitution, keeping both points.

Inequalities: linear in one or two variables and quadratic - $x^2 - x - 6 > 0$ gives $x < -2$ or $x > 3$ (outside the roots). Variation: direct $y = kx^n$, inverse $y = k/x^n$ (not sampled).

Check yourself

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Substitution and subject-twice rearrangement; triple expansions; all factorising; quadratic fractions; formal proof. Functions with domain, range, composite and inverse.

Line equations; curve families sketched; completing the square; differentiation with tangents and stationary points; all solving routes and inequality types; linear and quadratic n th terms.

Exercises 2.1·2.3

2.1 Solve $5x - 3 = 2x + 9$ [2]

2.2 Solve $x^2 - 3x - 10 = 0$ [2]

2.3 Solve $x^2 + 6x + 2 = 0$, giving exact roots [3]

Exercises 2.1·2.3

2.1 Solve $5x - 3 = 2x + 9$ [2]

2.1 $3x = 12$, so $x = 4$.

2.2 Solve $x^2 - 3x - 10 = 0$ [2]

2.2 $(x - 5)(x + 2) = 0$: $x = 5$ or -2 .

2.3 Solve $x^2 + 6x + 2 = 0$, giving exact roots [3]

2.3 $x = \frac{-6 \pm \sqrt{36 - 8}}{2} = \frac{-6 \pm \sqrt{28}}{2} = -3 \pm \sqrt{7}.$

Exercises 2.4·3.1

2.4 Solve the pair $y = 2x + 1$, $y = x^2 - 2$ [4]

2.5 Solve $\frac{x}{3} + 2 > \frac{x}{2}$ [3]

3.1 (*Original, 1E intersection family.*) A line $y = 2x + 5$ meets the curve $y = x^2 + 1$ at two points. (a) Show the x -coordinates satisfy $x^2 - 2x - 4 = 0$. (b) Find both intersection points, exact where possible. [5]

Exercises 2.4·3.1

2.4 Solve the pair $y = 2x + 1$, $y = x^2 - 2$ [4]

2.4 $x^2 - 2 = 2x + 1$, so $x^2 - 2x - 3 = 0 = (x - 3)(x + 1)$: $x = 3$ or -1 , giving $(3, 7)$ and $(-1, -1)$.

2.5 Solve $\frac{x}{3} + 2 > \frac{x}{2}$ [3]

2.5 Multiply by 6: $2x + 12 > 3x$, so $x < 12$.

3.1 (*Original, 1E intersection family.*) A line $y = 2x + 5$ meets the curve $y = x^2 + 1$ at two points. (a) Show the x -coordinates satisfy $x^2 - 2x - 4 = 0$. (b) Find both intersection points, exact where possible. [5]

3.1 (a) $2x + 5 = x^2 + 1$ gives $x^2 - 2x - 4 = 0$. (b) $x = \frac{2 \pm \sqrt{4 + 16}}{2} = 1 \pm \sqrt{5}$; points $(1 + \sqrt{5}, 7 + 2\sqrt{5})$ and $(1 - \sqrt{5}, 7 - 2\sqrt{5})$.

Exercises 3.2·3.3

3.2 (*Original.*) A rectangular field's length exceeds its width by 5 m; its area is 84 m^2 . Form and solve a quadratic to find both dimensions. [4]

3.3 (*Original, 2E inequality family.*) Solve and represent on a number line: $-7 \leq 3x+2 < 6$. [3]

Exercises 3.2·3.3

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3.2 Width w : $w(w + 5) = 84$, so $w^2 + 5w - 84 = 0 = (w + 12)(w - 7)$: width 7 m, length 12 m (reject -12).

3.3 (*Original, 2E inequality family.*) Solve and represent on a number line: $-7 \leq 3x + 2 < 6$. [3]

3.3 Subtract 2 throughout: $-9 \leq 3x < 4$; divide by 3: $-3 \leq x < \frac{4}{3}$; closed circle at -3 , open at $\frac{4}{3}$.

Local review guidance: substitution shown before expansion; exact roots and both intersection points; context rejections justified. This is authored classroom guidance.