

Exercise walk-through

2.4

Differentiation

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1. Unit 3.2 at a glance

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Core A1-A26: notation through graphs, equations, inequalities and sequences.

Extension additions: subject-twice rearrangement, triple expansions, all quadratic factorising, fractional powers, quadratic fractions, formal proof, the full functions toolkit, line equations, exponential and trig graphs, differentiation with tangents and stationary points, completing the square, variation, all simultaneity, quadratic inequalities, quadratic sequences.

Must-know: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $a(x + b)^2 + c$ reads the turning point; $y - y_1 = m(x - x_1)$; $\frac{d}{dx}(kx^n) = knx^{(n-1)}$; the tangent's gradient IS the gradient at the point.

8. Differentiation

Differentiation

$d/dx(kx^n) = knx^{(n-1)}$, term by term. The gradient at a point is the tangent's gradient: $y = x^2$ at $x = 2$ has tangent $y = 4x - 4$; the 2E specimen's family is $dy/dx = 15$ with tangent $y = 15x - 36$.

Stationary points solve $dy/dx = 0$, classified as maxima, minima or points of inflection: $y = x^3 - 12x + 5$ has maximum $(-2, 21)$, minimum $(2, -11)$, and tangent $y = 15x - 49$ at $x = 3$.

Check yourself

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Substitution and subject-twice rearrangement; triple expansions; all factorising; quadratic fractions; formal proof. Functions with domain, range, composite and inverse.

Line equations; curve families sketched; completing the square; differentiation with tangents and stationary points; all solving routes and inequality types; linear and quadratic n th terms.

Exercises 2.1·2.3

2.1 Differentiate $y = x^3 - 12x + 5$ [2]

2.2 Differentiate $y = 4x^5 + 3x^2 - 7$ [2]

2.3 Find the gradient of $y = x^2$ at $x = 2$ and the tangent's equation [3]

Exercises 2.1·2.3

2.1 Differentiate $y = x^3 - 12x + 5$ [2]

2.1 $3x^2 - 12$.

2.2 Differentiate $y = 4x^5 + 3x^2 - 7$ [2]

2.2 $20x^4 + 6x$.

2.3 Find the gradient of $y = x^2$ at $x = 2$ and the tangent's equation [3]

2.3 $\frac{dy}{dx} = 2x$, so $m = 4$; $y - 4 = 4(x - 2)$ gives $y = 4x - 4$.

Exercises 2.4·3.2

2.4 Find the stationary points of $y = x^3 - 12x + 5$ [3]

3.1 (*Original, 2E tangent family.*) The curve $y = x^3 - 12x + 5$ has $\frac{dy}{dx} = 3x^2 - 12$. (a) Verify that the gradient at $x = 3$ is 15. (b) Find the equation of the tangent at $(3, -4)$. [4]

3.2 (*Original.*) A curve has $y = 2x^3 + 9x^2 - 24x + 1$. (a) Find $\frac{dy}{dx}$. (b) Find both stationary points' coordinates. (c) Classify each from the sketch shape. [6]

Exercises 2.4·3.2

2.4 Find the stationary points of $y = x^3 - 12x + 5$ [3]

2.4 $x = \pm 2$: maximum $(-2, 21)$, minimum $(2, -11)$.

3.1 (*Original, 2E tangent family.*) The curve $y = x^3 - 12x + 5$ has $\frac{dy}{dx} = 3x^2 - 12$. (a) Verify that the gradient at $x = 3$ is 15. (b) Find the equation of the tangent at $(3, -4)$. [4]

3.1 (a) $3(3)^2 - 12 = 27 - 12 = 15$. ✓ (b) $y + 4 = 15(x - 3)$, so $y = 15x - 49$.

3.2 (*Original.*) A curve has $y = 2x^3 + 9x^2 - 24x + 1$. (a) Find $\frac{dy}{dx}$. (b) Find both stationary points' coordinates. (c) Classify each from the sketch shape. [6]

3.2 (a) $6x^2 + 18x - 24$. (b) $6x^2 + 18x - 24 = 0$ gives $x^2 + 3x - 4 = 0$, so $(x + 4)(x - 1) = 0$: $x = -4$ and $x = 1$. Points: at $x = -4$, $y = -128 + 144 + 96 + 1 = 113$; at $x = 1$, $y = 2 + 9 - 24 + 1 = -12$. (c) Positive cubic: maximum $(-4, 113)$, minimum $(1, -12)$.

Exercise 3.3

3.3 (*Original, specification line not sampled: point of inflection.*) For $y = x^3$, explain why $x = 0$ is a stationary point but neither a maximum nor a minimum. [3]

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3.3 $\frac{dy}{dx} = 3x^2 = 0$ at $x = 0$ (stationary), but the gradient is positive on BOTH sides of $x = 0$ – the curve keeps rising, so the point is a point of inflection, not a turning point.

Local review guidance: differentiation before any substitution; exact coordinates; classification argues from gradient signs or shape. This is authored classroom guidance.