

Exercise walk-through

2.3

Lines, curves and inequalities

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1. Unit 3.2 at a glance

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Core A1-A26: notation through graphs, equations, inequalities and sequences.

Extension additions: subject-twice rearrangement, triple expansions, all quadratic factorising, fractional powers, quadratic fractions, formal proof, the full functions toolkit, line equations, exponential and trig graphs, differentiation with tangents and stationary points, completing the square, variation, all simultaneity, quadratic inequalities, quadratic sequences.

Must-know: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $a(x + b)^2 + c$ reads the turning point; $y - y_1 = m(x - x_1)$; $\frac{d}{dx}(kx^n) = knx^{(n-1)}$; the tangent's gradient IS the gradient at the point.

6. Lines and curves

Lines and curves

Lines: $m = (y_2 - y_1)/(x_2 - x_1)$; $y - y_1 = m(x - x_1)$; through (1, 7) and (2, 9): $y = 2x + 5$. Perpendicular gradients multiply to -1 (not sampled).

Curves: linear, quadratic, cubic, reciprocal; exponential $y = k^x$ and trig graphs (not sampled); turning points by completing the square: $x^2 - 6x + 5 = (x - 3)^2 - 4$, minimum (3, -4). Gradients and areas estimated from graphs.

7. Curve families and turning points

Curve families and turning points

Sketch and interpret linear, quadratic, cubic and reciprocal graphs; Extension adds exponential $y = k^x$ and the trigonometric graphs in degrees (specification lines, not sampled by the specimens).

Roots, intercepts and turning points read from sketches; turning points DEDUCED by completing the square: $x^2 - 6x + 5 = (x - 3)^2 - 4$ has its minimum at (3, -4). Gradients and areas under graphs estimated by tangents and trapezia.

Exercises 2.1·2.3

2.1 Find the equation of the line through $(1, 7)$ and $(2, 9)$ [3]

2.2 Find the equation of the line through $(0, -3)$ with gradient 4 [2]

2.3 Write down the gradient perpendicular to $m = \frac{2}{5}$ [1]

Exercises 2.1·2.3

2.1 Find the equation of the line through $(1, 7)$ and $(2, 9)$ [3]

2.1 $m = 2$, so $y = 2x + 5$.

2.2 Find the equation of the line through $(0, -3)$ with gradient 4 [2]

2.2 $y = 4x - 3$.

2.3 Write down the gradient perpendicular to $m = \frac{2}{5}$ [1]

2.3 $-\frac{5}{2}$.

Exercises 2.4·3.1

2.4 Solve $x^2 - x - 6 > 0$ [3]

2.5 Solve $-7 \leq 3x + 2 < 6$ (the 2E specimen's answer family) [3]

3.1 (*Original.*) A line joins $A(-1, 3)$ to $B(3, 11)$. (a) Find its equation. (b) Find the equation of the perpendicular line through the midpoint of AB . [5]

Exercises 2.4·3.1

2.4 Solve $x^2 - x - 6 > 0$ [3]

2.4 $(x - 3)(x + 2) > 0$: positive outside the roots, so $x < -2$ or $x > 3$.

2.5 Solve $-7 \leq 3x + 2 < 6$ (the 2E specimen's answer family) [3]

2.5 Subtract 2 throughout: $-9 \leq 3x < 4$; divide by 3: $-3 \leq x < \frac{4}{3}$.

3.1 (*Original.*) A line joins $A(-1, 3)$ to $B(3, 11)$. (a) Find its equation. (b) Find the equation of the perpendicular line through the midpoint of AB . [5]

3.1 (a) $m = \frac{11-3}{3-(-1)} = 2$, so $y = 2x + 5$. (b) Midpoint $(1, 7)$; perpendicular gradient $-\frac{1}{2}$:
 $y - 7 = -\frac{1}{2}(x - 1)$, so $y = -\frac{1}{2}x + \frac{15}{2}$.

Exercises 3.2·3.3

3.2 (*Original, specification line not sampled: completing the square.*) Write $x^2 + 6x + 2$ in the form $(x + a)^2 + b$, hence state the minimum point and solve $x^2 + 6x + 2 = 0$ giving exact roots (the 1E specimen's answer family). [5]

3.3 (*Original.*) Sketch or describe the graph of $y = 2^x$, giving the y -intercept and the behaviour as x grows and as $x \rightarrow -\infty$. [3]

Exercises 3.2·3.3

3.2 (*Original, specification line not sampled: completing the square.*) Write $x^2 + 6x + 2$ in the form $(x + a)^2 + b$, hence state the minimum point and solve $x^2 + 6x + 2 = 0$ giving exact roots (the 1E specimen's answer family). [5]

3.2 $x^2 + 6x + 2 = (x + 3)^2 - 7$. Minimum $(-3, -7)$; roots: $(x + 3)^2 = 7$, so $x = -3 \pm \sqrt{7}$.

3.3 (*Original.*) Sketch or describe the graph of $y = 2^x$, giving the y -intercept and the behaviour as x grows and as $x \rightarrow -\infty$. [3]

3.3 Passes through $(0, 1)$; rises ever faster as x grows; as $x \rightarrow -\infty$ the curve approaches the x -axis (never reaches 0).

Local review guidance: gradient then point-slope form; exact roots; inequality solution sets stated with the correct strict/non-strict signs. This is authored classroom guidance.