

Exercise walk-through

2.2

Functions and proof

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1. Unit 3.2 at a glance

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Core A1-A26: notation through graphs, equations, inequalities and sequences.

Extension additions: subject-twice rearrangement, triple expansions, all quadratic factorising, fractional powers, quadratic fractions, formal proof, the full functions toolkit, line equations, exponential and trig graphs, differentiation with tangents and stationary points, completing the square, variation, all simultaneity, quadratic inequalities, quadratic sequences.

Must-know: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $a(x + b)^2 + c$ reads the turning point; $y - y_1 = m(x - x_1)$; $d/dx(kx^n) = knx^{(n-1)}$; the tangent's gradient IS the gradient at the point.

5. Functions

Functions

$f(x)$ maps each input to one output; domain and range stated. Composite $fg(x) = f(g(x))$ - g first: the 2E specimen builds $fg(x) = (x - 5)^2 + 1$ and solves $fg(x) = 5$ to $x = 3$ (or 7).

Inverse by swap-and-rearrange: the 1E specimen's $f^{-1}(x) = x/3$ comes from $f(x) = 3x$. Check $f(f^{-1}(x)) = x$.

Check yourself

Check yourself

Substitution and subject-twice rearrangement; triple expansions; all factorising; quadratic fractions; formal proof. Functions with domain, range, composite and inverse.

Line equations; curve families sketched; completing the square; differentiation with tangents and stationary points; all solving routes and inequality types; linear and quadratic n th terms.

Exercises 2.1·2.3

2.1 For $f(x) = 3x - 5$, find $f(4)$ [1]

2.2 For $f(x) = 3x - 5$ and $g(x) = x^2$, find $fg(2)$ [2]

2.3 Find $f^{-1}(x)$ for $f(x) = 3x$ [2]

Exercises 2.1·2.3

2.1 For $f(x) = 3x - 5$, find $f(4)$ [1]

2.1 7.

2.2 For $f(x) = 3x - 5$ and $g(x) = x^2$, find $fg(2)$ [2]

2.2 $g(2) = 4$, then $f(4) = 7$.

2.3 Find $f^{-1}(x)$ for $f(x) = 3x$ [2]

2.3 $f^{-1}(x) = \frac{x}{3}$.

Exercises 2.4·3.2

2.4 State the domain and range of $f(x) = \sqrt{x - 2}$ [2]

3.1 (*Original, 2E composite family.*) $g(x) = x - 5$ and $f(x) = x^2 + 1$. (a) Find $fg(x)$. (b) Solve $fg(x) = 5$. [4]

3.2 (*Original, 1E inverse family.*) $f(x) = \frac{x}{3} + 2$. (a) Find $f^{-1}(x)$. (b) Verify that $f^{-1}(14) = 36$. [4]

Exercises 2.4·3.2

2.4 State the domain and range of $f(x) = \sqrt{x - 2}$ [2]

2.4 Domain $x \geq 2$; range $f(x) \geq 0$.

3.1 (*Original, 2E composite family.*) $g(x) = x - 5$ and $f(x) = x^2 + 1$. (a) Find $fg(x)$. (b) Solve $fg(x) = 5$. [4]

3.1 (a) $fg(x) = (x - 5)^2 + 1$. (b) $(x - 5)^2 = 4$, so $x = 3$ or $x = 7$.

3.2 (*Original, 1E inverse family.*) $f(x) = \frac{x}{3} + 2$. (a) Find $f^{-1}(x)$. (b) Verify that $f^{-1}(14) = 36$. [4]

3.2 (a) $y = \frac{x}{3} + 2 \Rightarrow x = 3(y - 2)$, so $f^{-1}(x) = 3x - 6$. (b) $f^{-1}(14) = 42 - 6 = 36$. ✓

Exercise 3.3

3.3 (*Original, 2E proof family.*) Prove that the difference of the squares of any two consecutive odd numbers is always a multiple of 8. [4]

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3.3 Consecutive odd numbers: $2n + 1$ and $2n - 1$. Their squares differ by $(2n + 1)^2 - (2n - 1)^2 = (4n^2 + 4n + 1) - (4n^2 - 4n + 1) = 8n$, a multiple of 8 for every integer n .

Local review guidance: composite order – g first inside f ; inverses with domains; proofs end with the conclusion in words. This is authored classroom guidance.