Geometry and measures

OxfordAQA International GCSE Mathematics 9260 unit 3.3: geometry and measures, Core and Extension

Geometry and measures

- use angle facts for lines, polygons and parallel lines, and give reasons;
- use congruence, similarity and circle theorems (Extension);
- use **bearings**, **constructions**, **loci** and units;

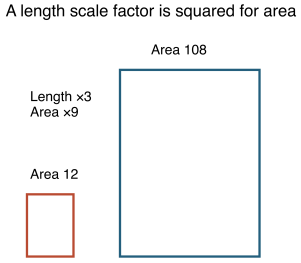
Geometry and measures (2)

- find areas, surface areas and volumes, including sectors and cones;
- use Pythagoras and trigonometry in 2D and 3D, and the sine and cosine rules (Extension);
- carry out transformations, and use vectors and matrices (Extension).

Angles, polygons and symmetry

- Angles at a point add up to $360°$; angles on a straight line add up to $180°$.
- **Vertically opposite** angles are equal.
- With parallel lines, **alternate angles** are equal, **corresponding angles** are equal, and co-interior angles add up to $180°$. Write these names, never "Z angles" or "F angles".
- The angles in a triangle add up to $180°$.

Figure: Angles, polygons and symmetry



- Length scale factor 3
- Area scale factor 9: $12 \times 9 = 108$

Congruence, similarity and circle theorems

- The angle at the centre is twice the angle at the **circumference** on the same **arc**.
- The angle in a **semicircle** is $90°$.
- Angles in the same segment are equal.

Congruence, similarity and circle theorems (2)

- Opposite angles of a **cyclic quadrilateral** add up to $180°$.
- A tangent is perpendicular to the **radius** at the point where it touches.
- Two tangents from a point outside the circle are equal in length.

Congruence, similarity and circle theorems (3)

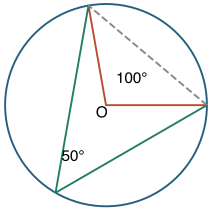
- The perpendicular from the centre to a **chord** bisects the **chord**.
- The angle between a tangent and a chord equals the angle in the **alternate segment** .
- $\angle AOC = 2 \times 35° = 70°$ (the angle at the centre is twice the angle at the **circumference**). The reflex angle AOC is $360° - 70° = 290°$.

Congruence, similarity and circle theorems (4)

- $\angle ABC = 180° - 35° = 145°$ (opposite angles of a **cyclic quadrilateral** add up to $180°$).
- Check: $\angle ABC$ is half the reflex angle at the centre, $\frac{1}{2} \times 290° = 145°$.

Figure: Congruence, similarity and circle theorems

The centre angle is twice the circumference angle



- Same **arc**: 100° at the centre
- and 50° at the **circumference**

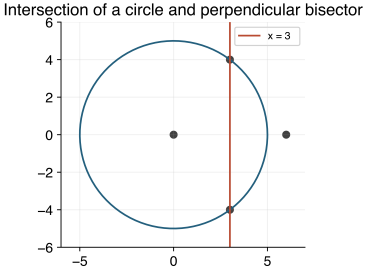
3D shapes, constructions and bearings

- The bearing of A from B is $070° + 180° = 250°$.
- At B, A is on $250°$ and C is on $160°$, so angle $ABC = 250° - 160° = 90°$.
- the **perpendicular bisector** of a line segment;

3D shapes, constructions and bearings (2)

- the perpendicular to a line from a point, or at a point on it;
- the **angle bisector** ;
- an angle of $60°$.

Figure: 3D shapes, constructions and bearings



- Circle: points 5 from O
- x = 3 is the **perpendicular bisector**; they meet at (3, ±4)

Units, mensuration and similarity ratios

- **Arc** $= \frac{150}{360} \times 2\pi \times 6 = 5\pi = 15.7$ cm, so the perimeter is $5\pi + 12 = 27.7$ cm.
- Area $= \frac{150}{360} \times \pi \times 36 = 15\pi = 47.1$ cm^2.
- Slant height $l = \sqrt{5^2 + 12^2} = 13$ cm.
- $V = \frac{1}{3} \times \pi \times 25 \times 12 = 100\pi$ cm^3. Curved surface $= \pi \times 5 \times 13 = 65\pi$ cm^2.

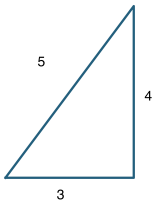
Pythagoras, trigonometry and 3D problems

- $c^2 = 9^2 + 11^2 - 2 \times 9 \times 11 \times \cos 72° = 202 - 61.2 = 140.8$, so $c = 11.9$ (3 s.f.).
- $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{121 + 140.8 - 81}{2 \times 11 \times 11.87} = 0.693$, so $A = 46.2°$.
- Check: $B = 180° - 72° - 46.2° = 61.8°$, and the smallest angle A is opposite the shortest side a.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \quad \cos\theta = \frac{\text{adj}}{\text{hyp}}, \quad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Figure: Pythagoras, trigonometry and 3D problems

Label the known sides before choosing a rule



- Label opposite, adjacent, hypotenuse first
- Here 3, 4 and 5

Transformations with matrices and vectors

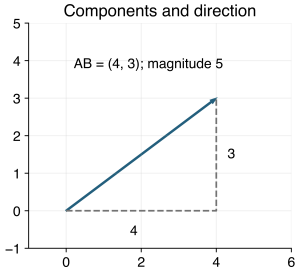
- The first transformation goes on the right:
  $\mathbf{RM} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.
- This is a reflection in the y-axis.
- Check with $(2, 1)$: the reflection gives $(1, 2)$, the rotation gives $(-2, 1)$, and a reflection in the y-axis also gives $(-2, 1)$.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

Transformations with matrices and vectors (2)

- Multiply out: $3x + y = 10$ and $x + 2y = 5$.
- From the first, $y = 10 - 3x$. Substitute: $x + 2(10 - 3x) = 5$, so $-5x = -15$ and $x = 3$.
- Then $y = 1$. Check: $3 \times 3 + 1 = 10$.

Figure: Transformations with matrices and vectors



- AB $= (4, 3)$
- Magnitude $\sqrt{(4^2 + 3^2)} = 5$

Sources for this lesson

- OxfordAQA, *International GCSE Mathematics (9260) specification*, for first assessment 2018, unit 3.3.
- OxfordAQA, 9260 specimen Papers 1 and 2 (Extension), 2016 (adapted).
- Diogenes Laertius, *Lives of the Eminent Philosophers*, book 1, which records Thales' measurement.

Common errors

- Giving "Z angles" instead of "**alternate angles**" as a reason.
- Using the interior angle sum $(n-2)\times 180^\circ$ for exterior angles.
- Forgetting the two radii in the perimeter of a **sector**.

Common errors (2)

- Multiplying a volume by k or k^2 instead of k^3.
- Measuring a bearing anticlockwise, or from the wrong point.
- Leaving the calculator in radians.
- Multiplying matrices in the order the transformations happen; the first one goes on the right.

Key terms

- **Congruent**: exactly the same shape and size.
- **Similar**: the same shape, with all lengths multiplied by the same scale factor.
- **Cyclic quadrilateral**: a quadrilateral with all four vertices on a circle.

Key terms (2)

- **Loci**: sets of points that obey a rule.
- **Frustum**: the part of a cone left when the top is cut off parallel to the base.
- **Identity matrix**: the **matrix** that leaves every point unchanged.

Check yourself

- 1.
- Find the interior angle of a regular hexagon. 2.
- Q is on a bearing of $300°$ from P.
- Find the bearing of P from Q. 3.