# Algebra

OxfordAQA International GCSE Mathematics 9260 unit 3.2: algebra, Core and Extension

# Algebra

- substitute into formulae and rearrange them;
- expand brackets, factorise, and simplify algebraic fractions;
- write algebraic proofs;

# Algebra (2)

- use functions, straight-line graphs and the graphs of other functions;
- differentiate to find gradients, tangents and stationary points (Extension);
- solve equations, simultaneous equations and inequalities;
- find the $n$th term of linear and quadratic sequences.

# Notation, substitution and formulae

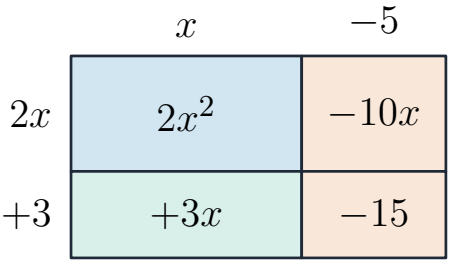
- Collect the terms with $a$ on the left: $ax + 2a = 5xt - 3$.
- Factorise: $a(x + 2) = 5xt - 3$.
- Divide: $a = \dfrac{5xt - 3}{x + 2}$.

# Expanding, factorising and algebraic fractions

- (a) $\dfrac{x^2-9}{x^2+x-6} = \dfrac{(x-3)(x+3)}{(x+3)(x-2)} = \dfrac{x-3}{x-2}$.
- (b) The common denominator is $(x+2)(x-1)$. Keep the second numerator in a bracket so that the minus sign applies to both terms:
$$\frac{3(x-1)-(x+2)}{(x+2)(x-1)} = \frac{2x-5}{(x+2)(x-1)}.$$

$$(x+2)(x-3)(x+1) = (x^2-x-6)(x+1) = x^3-7x-6$$

# Figure: Expanding, factorising and algebraic fractions




$-10x + 3x = -7x$ ✓

$3 \times (-5) = -15$ ✓

$(2x + 3)(x - 5)$

- (2x + 3)(x − 5): the x-terms add to −7x
- The corner term is −15

# Algebraic proof

- $n^2 + n = n(n + 1)$, the product of two **consecutive integers**.
- One of any two consecutive integers is even, so $n(n + 1)$ is even.
- Even $+9$ is odd, so $n^2 + n + 9$ is odd.

# Algebraic proof (2)

- Let the odd numbers be $2n-1$ and $2n+1$.
- $(2n+1)^2-(2n-1)^2=(4n^2+4n+1)-(4n^2-4n+1)=8n$, which is a multiple of 8.

# Functions

- The **domain** is the set of inputs; the **range** is the set of outputs.
- A **composite function** $fg(x)$ means $f(g(x))$: apply $g$ first.
- An **inverse function** $f^{-1}$ undoes $f$. Write $y = f(x)$, make $x$ the subject, then swap $x$ and $y$. If $f(x) = 3x$, then $f^{-1}(x) = \frac{x}{3}$.

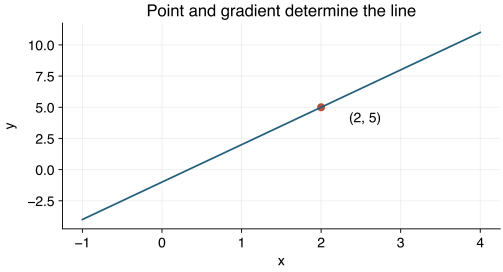
# Functions (2)

- $fg(x) = f(x-5) = (x-5)^2 + 1$.
- $(x-5)^2 + 1 = 5$, so $(x-5)^2 = 4$ and $x - 5 = \pm 2$.
- $x = 3$ or $x = 7$.

# Straight-line graphs and coordinates

- Coordinates $(x, y)$ can be in any of the four **quadrants** .
- The line $y = mx + c$ has gradient $m$ and crosses the $y$-axis at $(0, c)$.
- (Extension) To find the equation of a line through two points, find the gradient first.
- Through $(1, 7)$ and $(2, 9)$: $m = \frac{9-7}{2-1} = 2$, then $7 = 2 \times 1 + c$, so $c = 5$ and $y = 2x + 5$.

# Figure: Straight-line graphs and coordinates




- Gradient 3 through (2, 5)
- $y - 5 = 3(x - 2)$, so $y = 3x - 1$

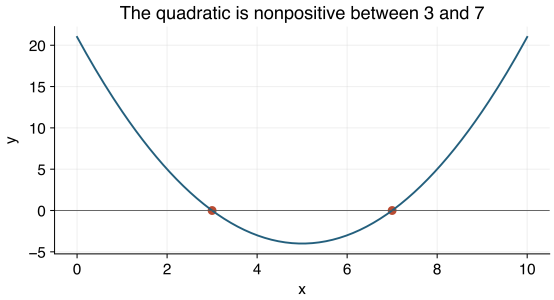
# Curves and their graphs

- quadratic, $y = ax^2 + bx + c$: a U shape if $a > 0$, an upside-down U if $a < 0$
- **cubic** , $y = ax^3 + \ldots$: an S-shaped curve
- reciprocal, $y = \frac{k}{x}$: two curves that approach the axes but never touch them

# Curves and their graphs (2)

- (Extension) exponential, $y = k^x$ with $k > 0$: crosses the $y$-axis at $(0, 1)$
- (Extension) $y = \sin x$, $y = \cos x$ and $y = \tan x$, for $x$ in degrees.

# Figure: Curves and their graphs




- $x^2 - 10x + 21 = (x - 3)(x - 7)$
- Below the axis for $3 \leq x \leq 7$

# Differentiation

- $\frac{dy}{dx} = 3x^2 - 12 = 0$, so $x^2 = 4$ and $x = \pm 2$.
- At $x = -2$: $y = -8 + 24 + 5 = 21$. The gradient goes from $+$ to $-$, so $(-2, 21)$ is a maximum.
- At $x = 2$: $y = 8 - 24 + 5 = -11$. The gradient goes from $-$ to $+$, so $(2, -11)$ is a minimum.
- At $x = 3$: $y = 27 - 36 + 5 = -4$ and the gradient is $3 \times 9 - 12 = 15$. The **tangent** is $y + 4 = 15(x - 3)$, so $y = 15x - 49$.

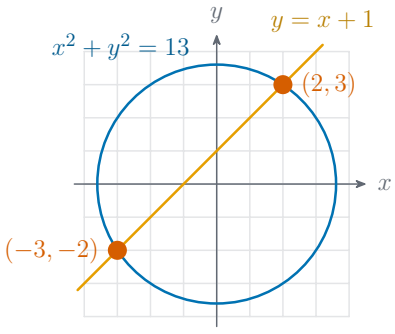
$$y = kx^n \implies \frac{dy}{dx} = knx^{n-1}$$

# Equations and inequalities

- Substitute: $x^2 + (x+1)^2 = 13$, so $2x^2 + 2x - 12 = 0$ and $x^2 + x - 6 = 0$.
- $(x+3)(x-2) = 0$, so $x = 2$ or $x = -3$.
- Use $y = x + 1$: the solutions are $x = 2, y = 3$ and $x = -3, y = -2$.

$$x = \frac{-6 \pm \sqrt{36 - 8}}{2} = \frac{-6 \pm \sqrt{28}}{2} = -3 \pm \sqrt{7}$$

# Figure: Equations and inequalities

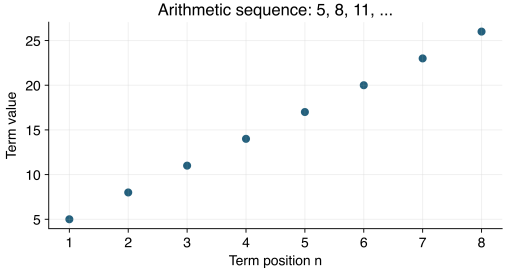



- y = x + 1 meets $x^2 + y^2 = 13$ twice
- Answers in pairs: (2, 3) and (−3, −2)

# Sequences

- First differences: 4, 8, 12, 16. **Second difference**: 4, so $a = 2$.
- Subtract $2n^2$ (2, 8, 18, 32, 50) from the terms: 3, 1, $-1$, $-3$, $-5$. This is linear, with $n$th term $-2n + 5$.
- So the $n$th term is $2n^2 - 2n + 5$. Check $n = 4$: $32 - 8 + 5 = 29$.

# Figure: Sequences




- Add 3 each time: **nth term** 3n + 2
- The eighth term is 26

# Sources for this lesson

- OxfordAQA, *International GCSE Mathematics (9260) specification*, for first assessment 2018, unit 3.2.
- OxfordAQA, 9260 specimen Papers 1 and 2 (Extension), 2016 (adapted).
- F. Viète, *In artem analyticem isagoge* (Tours, 1591).

# Common errors

- Writing $(x+4)^2 = x^2 + 16$; it is $x^2 + 8x + 16$.
- Losing the minus sign on the second term when subtracting fractions.
- Cancelling single terms instead of whole factors.

# Common errors (2)

- Using numerical examples as a **proof**.
- Working out $fg(x)$ by applying $f$ first.
- Forgetting to reverse the inequality sign when dividing by a negative number.

# Common errors (3)

- Giving the solutions of simultaneous equations as unpaired lists of $x$ and $y$ values.
- Finding a **stationary point**'s $x$-coordinate but not its $y$-coordinate.

# Key terms

- **Identity**: an equation that is true for every value of the variable.
- **Composite function**: one **function** applied after another.
- **Inverse function**: the function that undoes another function.

# Key terms (2)

- **Turning point**: a point where a curve changes from going up to going down, or the reverse.
- **Derivative**: the **gradient function** $\frac{dy}{dx}$.
- **Second difference**: the difference between consecutive first differences of a sequence.

# Check yourself

- 1.
- Find the value of $3p^2 - q$ when $p = -2$ and $q = 5$. 2.
- Expand and simplify $(2x - 3)(x + 4)$. 3.
- Factorise $x^2 - 2x - 15$. 4.