Number

OxfordAQA International GCSE Mathematics 9260 unit 3.1: number, Core and Extension

Number

- order numbers and calculate with integers, decimals, fractions and mixed numbers;
- use factors, primes, the HCF and the LCM;
- use powers, roots, **standard form** and surds;

Number (2)

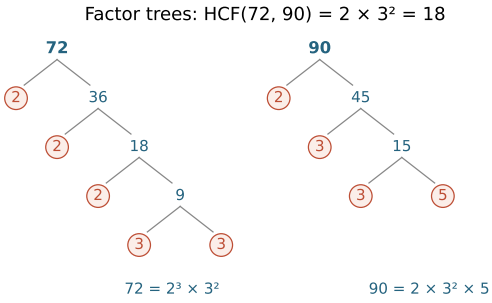
- round, and calculate with upper and lower bounds;
- use fractions, decimals and percentages, including reverse percentages and compound interest;
- solve problems with **ratio**, rates, proportion, and **exponential** growth and decay.

Ordering and the four operations

- An **integer** is a whole number: positive, negative or zero.
- Order integers, decimals and fractions on a **number line** using the symbols $=$, $\neq$, $<$ and $>$.
- To compare different forms, change them to the same form first: $\frac{3}{5} = 0.6$, so $\frac{3}{5} < 0.62$.
- To add or subtract fractions, use a common **denominator**.

$$2\tfrac{1}{3} + 1\tfrac{1}{4} = \tfrac{7}{3} + \tfrac{5}{4} = \tfrac{28}{12} + \tfrac{15}{12} = \tfrac{43}{12} = 3\tfrac{7}{12}$$

Figure: Ordering and the four operations



- $72 = 2^3 \times 3^2$ and $90 = 2 \times 3^2 \times 5$
- $\text{HCF} = 2 \times 3^2 = 18$

Factors, primes, HCF and LCM

- The **highest common factor** (HCF) is the largest number that divides both: take each **prime** they share, with the lower power.
- The **lowest common multiple** (LCM) is the smallest number in both times tables: take every prime that appears, with the higher power.
- $504 = 2^3 \times 3^2 \times 7$ and $588 = 2^2 \times 3 \times 7^2$.

Factors, primes, HCF and LCM (2)

- HCF $= 2^2 \times 3 \times 7 = 84$.
- LCM $= 2^3 \times 3^2 \times 7^2 = 8 \times 9 \times 49 = 3528$.

Indices, roots and surds

- The minus sign turns the fraction upside down: $\left(\frac{81}{16}\right)^{3/4}$.
- The **denominator** 4 of the power means a fourth root: $\sqrt[4]{\frac{81}{16}} = \frac{3}{2}$.
- The **numerator** 3 means cube: $\left(\frac{3}{2}\right)^3 = \frac{27}{8}$.

$$\sqrt{50} - \sqrt{18} + \sqrt{8} = 5\sqrt{2} - 3\sqrt{2} + 2\sqrt{2} = 4\sqrt{2}$$

Standard form, calculators and sets

- Multiply the numbers and add the powers: 24×10^3.
- 24 is not between 1 and 10, so write $24 = 2.4 \times 10^1$. The answer is 2.4×10^4.

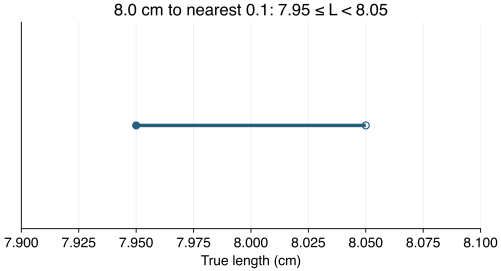
Rounding and bounds

- largest sum or product: **upper bound** $\times$ or $+$ upper bound
- largest difference: upper bound $-$ **lower bound**
- largest quotient: upper bound $\div$ lower bound.

Rounding and bounds (2)

- For a guaranteed answer, use the smallest floor and the largest space per person: **lower bound** 95.5 m^2, **upper bound** 5.15 m^2.
- $95.5 \div 5.15 = 18.5\ldots$
- Round down: 18 people. With 19, they could need $19 \times 5.15 = 97.85$ m^2, more than the floor may have.

Figure: Rounding and bounds



- 8.0 cm to the nearest 0.1 cm
- Bounds: $7.95 \leq L < 8.05$

Fractions, decimals and percentages as operators

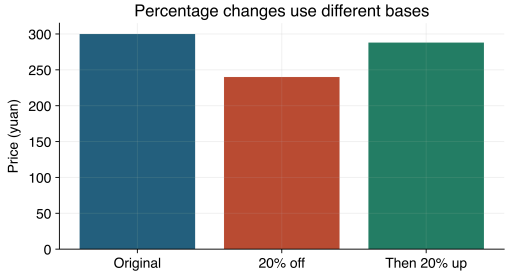
- Let $x = 0.2777\ldots$. Then $10x = 2.777\ldots$ and $100x = 27.777\ldots$.
- Subtract: $100x - 10x = 25$, so $90x = 25$ and $x = \frac{25}{90} = \frac{5}{18}$.

Percentage change and compound interest

- The sale price is 85% of the original, so original $\times 0.85 = 68$.
- Original $= 68 \div 0.85 = £80$.
- Adding 15% of £68 gives £78.20, which is wrong: the 15% is of the original price.

$$\text{percentage change} = \frac{\text{change}}{\text{original}} \times 100$$

Figure: Percentage change and compound interest



- 300 $\rightarrow$ 240 after 20% off
- Then 20% up gives 288: the second 20% is of 240

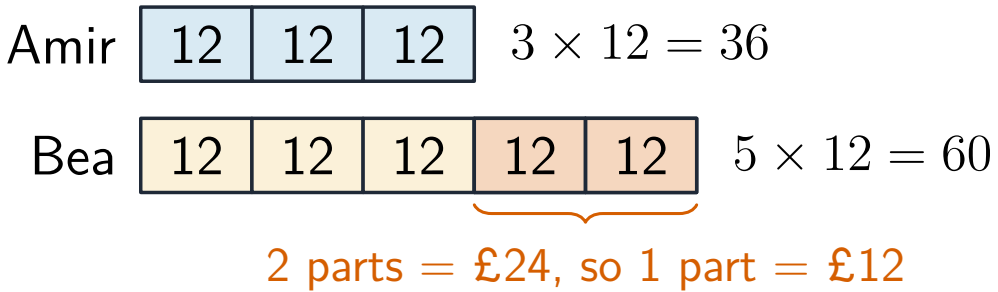
Ratio, rate and proportion

- The difference is $5 - 3 = 2$ parts, so 2 parts $= £24$ and 1 part $= £12$.
- Amir gets $3 \times 12 = £36$ and Bea gets $5 \times 12 = £60$. Check: $60 - 36 = 24$.
- $y = \dfrac{k}{x^2}$, so $5 = \dfrac{k}{4}$ and $k = 20$.

Ratio, rate and proportion (2)

- When $x = 4$: $y = \dfrac{20}{16} = 1.25$.
- Check: doubling x divides y by $2^2 = 4$, and $5 \div 4 = 1.25$.

Figure: Ratio, rate and proportion

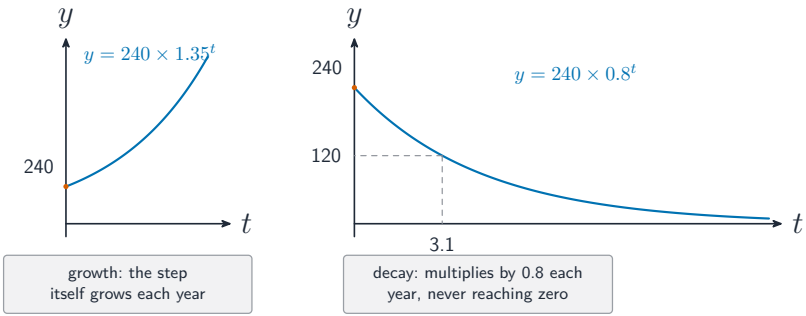


- The difference is 2 parts = £24
- One part = £12: Amir £36, Bea £60

Exponential growth and decay

- The value after t years is $V = 12\,000 \times 0.8^t$.
- Try values: $t = 7$ gives $12\,000 \times 0.8^7 = £2516.58$; $t = 8$ gives $12\,000 \times 0.8^8 = £2013.27$.
- So it is first worth less than £2500 after 8 years.

Figure: Exponential growth and decay



- Growth: $y = 240 \times 1.35^t$ rises ever faster
- Decay: $y = 240 \times 0.8^t$ falls towards zero

Sources for this lesson

- OxfordAQA, *International GCSE Mathematics (9260) specification*, for first assessment 2018, unit 3.1.
- OxfordAQA, 9260 specimen Papers 1 and 2 (Extension), 2016 (adapted).
- Archimedes, *The Sand Reckoner* (about 250 BC).

Common errors

- Adding fractions by adding the tops and the bottoms.
- Calling 1 a **prime** number.
- Writing 24×10^3 as a final answer in **standard form**.

Common errors (2)

- Thinking 4^{-2} is negative; it is $\frac{1}{16}$.
- Using the rounded value instead of a bound, or the wrong bound for a division.
- Finding a **reverse percentage** by taking the percentage of the new value.

Common errors (3)

- Using simple interest when the question says compound interest.
- Sharing in a **ratio** by dividing by one part of the ratio instead of the total number of parts.

Key terms

- **Prime factorisation**: a number written as a product of primes.
- **Standard form**: $A \times 10^n$ with $1 \leq A < 10$.
- **Surd**: a root that cannot be simplified to a whole number or fraction.

Key terms (2)

- **Upper bound**: the largest value a rounded number could have stood for.
- **Multiplier**: the number you multiply by to make a percentage change.
- **Inverse proportion**: when one quantity doubles, the other halves.

Check yourself

- 1.
- Work out $1\frac{2}{3} \times \frac{9}{10}$. 2.
- Find the HCF of 36 and 60. 3.
- Evaluate $16^{3/4}$. (Extension) 4.