4.2 Histograms, cumulative frequency and box plots

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS frequency density • box plot

Histograms, cumulative frequency and box plots

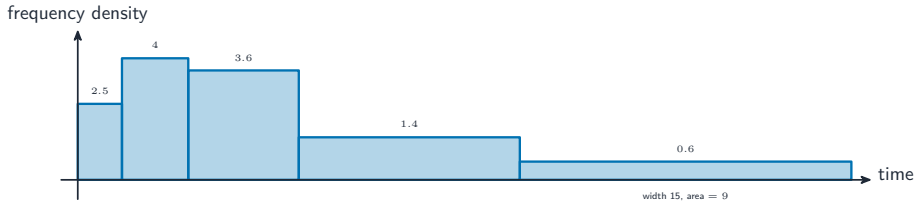
Objective

Draw histograms with unequal class intervals by frequency density, cumulative frequency diagrams at upper boundaries, and box plots (row S4 Extension).

1 Key knowledge and worked examples

Unequal histograms. Height is the frequency density = frequency over width; the AREA is the frequency: widths 2, 3, 5, 10, 15 holding 5, 12, 18, 14, 9 give heights 2.5, 4, 3.6, 1.4, 0.6 – the last bar looks tall only because it is wide (area $0.6 \times 15 = 9$).

CF and box plots. Plot cumulative totals at UPPER boundaries (the 1E specimen's correction); median at $n/2$, quartiles at $n/4$ and $3n/4$. Box plot: minimum, Q_1, median, Q_3, maximum – the box spans the IQR.



area = frequency: height is density = frequency over width

2 Practice

2.1 Classes of widths 4 and 8 hold frequencies 12 and 10. Find both densities. [2]

2.2 For 40 values, at which cumulative frequencies do the median and quartiles read? [3]

2.3 A box plot has minimum 3, Q_1 6, median 8, Q_3 11, maximum 17. Find the IQR and the range. [2]

2.4 A histogram bar of width 5 has density 6. Find its frequency. [2]

3 Exam-style questions

3.1 *(Original, 1E family.)* The times 60 workers spend on homework have cumulative frequencies 6, 17, 27, 42, 60 at upper boundaries 10, 15, 20, 25, 30 (hours). (a) State the plotting points. (b) Estimate the median. (c) Estimate the inter-quartile range. [5]

3.2 *(Original.)* A histogram has classes 0–10, 10–20, 20–40, 40–60 with frequencies 30, 20, 30, 10. (a) Compute the densities. (b) A student says the 20–40 class is largest because its bar is tallest. Correct the error using areas. [5]

3.3 *(Original, 2E 40-men family.)* A hall holds 96 m^2 (2 s.f.); safety needs 5.1 m^2 per person (1 d.p.). Using bounds, find the least whole number of people that must be able to fit. [3]

Solutions

2.1 $12/4 = 3$ and $10/8 = 1.25$.

2.2 Median at 20; quartiles at 10 and 30.

2.3 IQR $= 11 - 6 = 5$; range $= 17 - 3 = 14$.

2.4 $5 \times 6 = 30$.

3.1 (a) Points $(10, 6), (15, 17), (20, 27), (25, 42), (30, 60)$. (b) Worker 30: $20 + 5 \times \frac{3}{15} = 21$ hours. (c) Q_1: worker 15, $10 + 5 \times \frac{9}{11} = 14.1$; Q_3: worker 45, $25 + 5 \times \frac{3}{18} = 25.8$; IQR $= 11.7$ hours.

3.2 (a) Densities $30/10 = 3$, $20/10 = 2$, $30/20 = 1.5$, $10/20 = 0.5$. (b) By area the 0–10 class holds 30 and the 20–40 class also holds $1.5 \times 20 = 30$ – EQUAL, not larger; height compares frequencies only when widths match.

3.3 LB floor 95.5, UB per person 5.15: $\frac{95.5}{5.15} = 18.5\ldots$, so at least 19 people.

Local review guidance: densities before drawing, upper boundaries before plotting; interpolation inside the correct class; area comparisons for unequal widths. This is authored classroom guidance.