3.5 Transformations, vectors and matrices

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS vector • identity matrix

Transformations, vectors and matrices

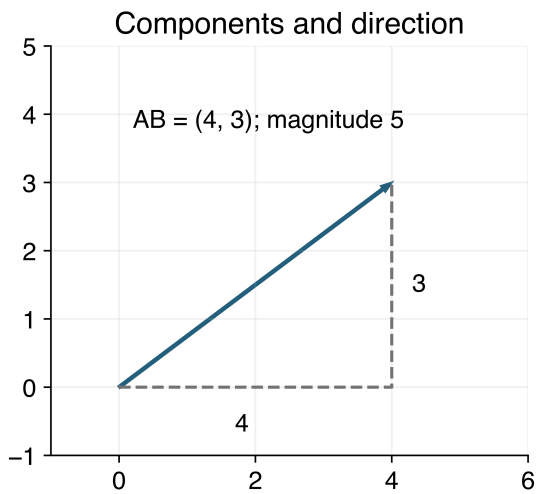
Objective

Transform 2D shapes including combined transformations and fractional/negative enlargements; vector notation and geometry; the 2×2 matrix programme with the identity and unit-square transformations (rows G21–G26 with the Extension additions).

1 Key knowledge and worked examples

Transformations. Single rotations, reflections, translations (by vector) and enlargements with preserved properties named; Extension: COMBINED transformations and fractional/negative scale factors (inverted, shrunken, opposite side of the centre).

Matrices. 2×2 multiplications and scalars; the identity I. Unit-square transformations: rotation 90° about O: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$; 180°: $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$; reflection in $y = -x$: $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ (the 1E specimen's matrix); enlargement factor k: $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$. Combinations multiply in order.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 Rotate $(-2, 4)$ by 90° anticlockwise about O [2]

2.2 Write the matrix for a reflection in the x-axis. [2]

2.3 Multiply $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ [2]

2.4 $\overrightarrow{AB} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$. Find $\overrightarrow{AC}$. [2]

2.5 Solve the matrix equation $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$ [3]

3 Exam-style questions

3.1 *(Original, 1E matrix family.)* The transformation T is represented by $\mathbf{M} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. (a) Find the image of the unit square's vertices under T. (b) Describe T fully. (c) Verify that $\mathbf{M}^2 = I$. [4]

3.2 *(Original, 1E matrix-solve family.)* Solve $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$, giving fractions where exact. [3]

3.3 *(Original, 2E vector-proof family.)* In triangle OCB, M is the midpoint of OB and N is the midpoint of CB. With $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OB} = \mathbf{b}$, show that $\overrightarrow{MN} = \frac{1}{2}\mathbf{c}$ and state what this proves about MN and OC. [4]

Solutions

2.1 $(x, y) \to (-y, x)$: $(-4, -2)$.

2.2 $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

2.3 $\begin{pmatrix} 10 \\ 5 \end{pmatrix}$.

2.4 $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$.

2.5 $3x + y = 10$, $x + 2y = 5$: $y = 10 - 3x$ gives $x + 20 - 6x = 5$, so $x = 3$, $y = 1$.

3.1 (a) $(0, 0) \to (0, 0)$; $(1, 0) \to (0, -1)$; $(0, 1) \to (-1, 0)$; $(1, 1) \to (-1, -1)$. (b) Reflection in the line $y = -x$. (c) $\mathbf{M}^2 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$. ✓

3.2 $2x + 3y = 0$ and $x + 4y = 5$: from the first $x = -\frac{3}{2}y$; then $-\frac{3}{2}y + 4y = 5$, so $\frac{5}{2}y = 5$, $y = 2$, $x = -3$: $(x, y) = (-3, 2)$.

3.3 $\overrightarrow{OM} = \frac{1}{2}\mathbf{b}$; $\overrightarrow{ON} = \frac{1}{2}(\mathbf{b} + \mathbf{c})$. So $\overrightarrow{MN} = \overrightarrow{ON} - \overrightarrow{OM} = \frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{c} - \frac{1}{2}\mathbf{b} = \frac{1}{2}\mathbf{c} = \frac{1}{2}\overrightarrow{OC}$: MN is parallel to OC and half its length.

Local review guidance: matrices multiplied in the order the transformations apply; exact coordinates and fractions; vector proofs end with the parallel-and-length statement. This is authored classroom guidance.