# 3.5 Transformations, vectors and matrices

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

**KEY WORDS** vector • identity matrix

## Transformations, vectors and matrices

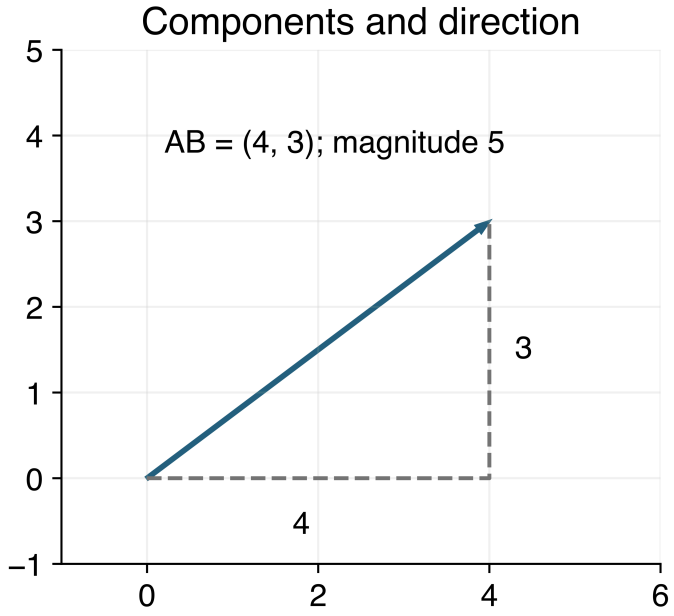
## Objective

Transform 2D shapes including combined transformations and fractional/negative enlargements; vector notation and geometry; the 2×2 matrix programme with the identity and unit-square transformations (rows G21–G26 with the Extension additions).

## 1 Key knowledge and worked examples

**Transformations.** Single rotations, reflections, translations (by vector) and enlargements with preserved properties named; Extension: COMBINED transformations and fractional/negative scale factors (inverted, shrunken, opposite side of the centre).

**Matrices.** 2×2 multiplications and scalars; the identity $I$. Unit-square transformations: rotation 90° about $O$: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$; 180°: $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$; reflection in $y = -x$: $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ (the 1E specimen's matrix); enlargement factor $k$: $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$. Combinations multiply in order.




*Original native-lesson illustration; labels belong to its worked example.*

## 2 Practice

**2.1** Rotate $(-2, 4)$ by $90°$ anticlockwise about $O$ [2]

**2.2** Write the matrix for a reflection in the $x$-axis. [2]

**2.3** Multiply $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ [2]

**2.4** $\overrightarrow{AB} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$. Find $\overrightarrow{AC}$. [2]

**2.5** Solve the matrix equation $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$ [3]

## 3 Exam-style questions

**3.1** *(Original, 1E matrix family.)* The transformation $T$ is represented by $\mathbf{M} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. (a) Find the image of the unit square's vertices under $T$. (b) Describe $T$ fully. (c) Verify that $\mathbf{M}^2 = I$. [4]

**3.2** *(Original, 1E matrix-solve family.)* Solve $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$, giving fractions where exact. [3]

**3.3** *(Original, 2E vector-proof family.)* In triangle $OCB$, $M$ is the midpoint of $OB$ and $N$ is the midpoint of $CB$. With $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OB} = \mathbf{b}$, show that $\overrightarrow{MN} = \frac{1}{2}\mathbf{c}$ and state what this proves about $MN$ and $OC$. [4]

## Solutions

**2.1** $(x, y) \to (-y, x)$: $(-4, -2)$.

**2.2** $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

**2.3** $\begin{pmatrix} 10 \\ 5 \end{pmatrix}$.

**2.4** $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$.

**2.5** $3x + y = 10$, $x + 2y = 5$: $y = 10 - 3x$ gives $x + 20 - 6x = 5$, so $x = 3$, $y = 1$.

**3.1** (a) $(0, 0) \to (0, 0)$; $(1, 0) \to (0, -1)$; $(0, 1) \to (-1, 0)$; $(1, 1) \to (-1, -1)$. (b) Reflection in the line $y = -x$. (c) $\mathbf{M}^2 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$. ✓

**3.2** $2x + 3y = 0$ and $x + 4y = 5$: from the first $x = -\frac{3}{2}y$; then $-\frac{3}{2}y + 4y = 5$, so $\frac{5}{2}y = 5$, $y = 2$, $x = -3$: $(x, y) = (-3, 2)$.

**3.3** $\overrightarrow{OM} = \frac{1}{2}\mathbf{b}$; $\overrightarrow{ON} = \frac{1}{2}(\mathbf{b} + \mathbf{c})$. So $\overrightarrow{MN} = \overrightarrow{ON} - \overrightarrow{OM} = \frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{c} - \frac{1}{2}\mathbf{b} = \frac{1}{2}\mathbf{c} = \frac{1}{2}\overrightarrow{OC}$: $MN$ is parallel to $OC$ and half its length.

Local review guidance: matrices multiplied in the order the transformations apply; exact coordinates and fractions; vector proofs end with the parallel-and-length statement. This is authored classroom guidance.