3.4 Trigonometry in 2D and 3D

Name: _____ Class: _____ Date: _____ **Total: 27 marks**

KEY WORDS sine rule • cosine rule

Objective

Build the skills to answer exam questions on **trigonometry** in 2D and 3D: right triangles, the sine rule, the cosine rule, the area of a triangle and angles in solids.

You must be able to:

- use Pythagoras and sine, cosine and tangent in right triangles
- use the sine rule and the cosine rule to find sides and angles
- find the area of a triangle with $\frac{1}{2}ab\sin C$
- find lengths and angles inside 3D shapes

1 Key knowledge and worked examples

Trigonometry

idea	rule
right triangle	$a^2+b^2=c^2$; $\sin\theta=\frac{\text{opp}}{\text{hyp}}$, $\cos\theta=\frac{\text{adj}}{\text{hyp}}$, $\tan\theta=\frac{\text{opp}}{\text{adj}}$
sine rule	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$: use it when you know a side and the angle opposite it
cosine rule	$a^2=b^2+c^2-2bc\cos A$: two sides and the angle between them, or three sides
angle from three sides	$\cos A=\frac{b^2+c^2-a^2}{2bc}$
area	$\frac{1}{2}ab\sin C$, with C the angle between sides a and b
3D	find a right triangle inside the solid; the angle between a line and a plane is the angle between the line and the line below it in the plane

Worked example 1: the sine rule. In triangle ABC, $a=8$ cm, $A=40°$ and $B=65°$. Find b.

1. Known: side a and its opposite angle A; angle B.
2. Rule: $\frac{b}{\sin B}=\frac{a}{\sin A}$.
3. Numbers: $b=\frac{8\sin 65°}{\sin 40°}=\frac{7.250}{0.6428}$.
4. Result: $b=11.3$ **cm**.

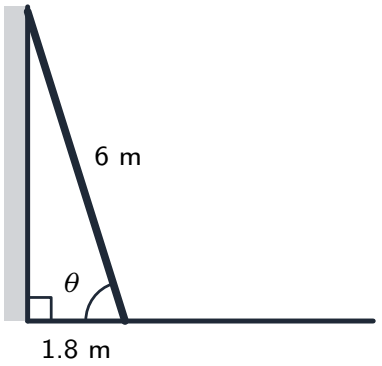
Worked example 2: an angle from three sides. A triangle has sides 5 cm, 7 cm and 8 cm. Find the angle opposite the 7 cm side.

1. Known: $a=7$, $b=5$, $c=8$.
2. Rule: $\cos A=\frac{b^2+c^2-a^2}{2bc}$.
3. Numbers: $\frac{25+64-49}{80}=\frac{40}{80}=0.5$.
4. Result: $A=60°$.

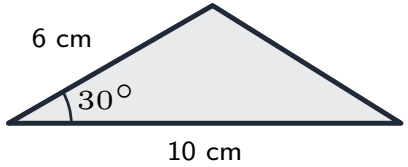
Answer shapes: give angles to 1 decimal place and lengths to 3 significant figures; keep full calculator values until the end.

2 Practice

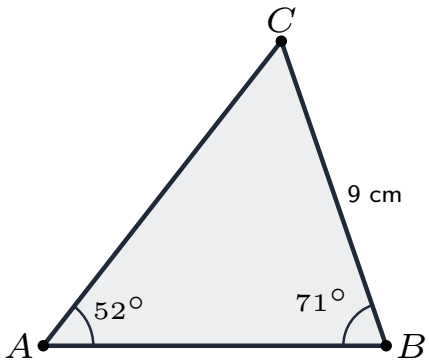
2.1 A ladder of length 6 m leans against a wall. The foot of the ladder is 1.8 m from the wall. Find the angle between the ladder and the ground. [2]



2.2 Find the area of the triangle. [2]

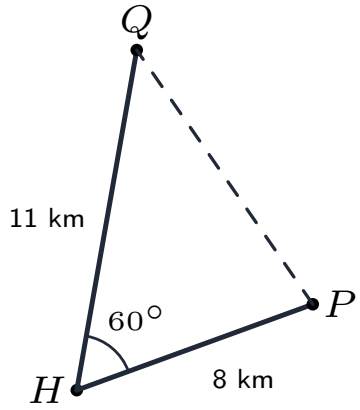


2.3 In triangle ABC, angle $A = 52°$, angle $B = 71°$ and $BC = 9$ cm. Find the length of AC. [2]

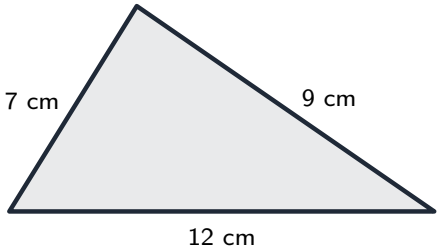


3 Exam-style questions

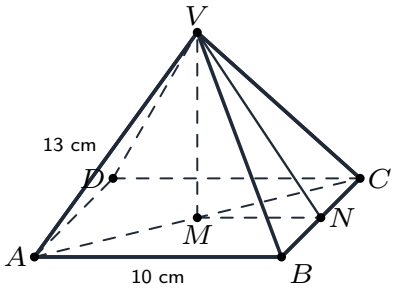
3.1 Two boats leave a harbour H. One sails 8 km to P. The other sails 11 km to Q. The angle PHQ is 60°. Find the distance PQ. [3]



3.2 A triangle has sides 7 cm, 9 cm and 12 cm. Find the largest angle of the triangle. [3]



3.3 $VABCD$ is a pyramid with a square base $ABCD$ of side 10 cm. The top V is directly above the centre M of the base. Each sloping edge, such as VA, is 13 cm. N is the midpoint of BC.



(a) Find the height VM of the pyramid. [3]

(b) Find the angle between the edge VA and the base. [3]

(c) Find the angle between the face VBC and the base. [4]

(d) Find angle AVB. Use $\frac{1}{2}ab\sin C$ to find the area of triangle VAB. Then find the total surface area of the pyramid. [5]

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Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

- $\cos\theta = \frac{1.8}{6} = 0.3$
- $\theta = \mathbf{72.5}^\circ$

2.2

- $\frac{1}{2} \times 6 \times 10 \times \sin 30^\circ$
- $= \mathbf{15}$ cm^2

2.3

- $\frac{AC}{\sin 71^\circ} = \frac{9}{\sin 52^\circ}$
- $AC = \mathbf{10.8}$ cm

3.1

- $PQ^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \cos 60^\circ$
- $= 64 + 121 - 88 = 97$
- $PQ = \mathbf{9.85}$ km

3.2

- the largest angle is opposite the 12 cm side
- $\cos\theta = \frac{49+81-144}{2\times7\times9} = -\frac{14}{126}$
- $\theta = \mathbf{96.4}^\circ$

3.3

(a) $AC = \sqrt{10^2 + 10^2} = 14.14$, so $AM = 7.071$; $VM^2 = 13^2 - 7.071^2 = 119$; $VM = \mathbf{10.9}$ cm

(b) the angle is VAM; $\cos VAM = \frac{7.071}{13}$; $\mathbf{57.0}^\circ$

(c) the angle is VNM, with $MN = 5$ cm; triangle VMN has a right angle at M; $\tan VNM = \frac{10.91}{5}$; $\mathbf{65.4}^\circ$

(d) $\cos AVB = \frac{13^2+13^2-10^2}{2\times13\times13} = \frac{238}{338}$; $AVB = 45.2^\circ$; area $= \frac{1}{2} \times 13 \times 13 \times \sin 45.2^\circ = 60.0$ cm^2; total $= 4 \times 60 + 10^2 = \mathbf{340}$ cm^2