3.2 Circle theorems with proof

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS proof

Circle theorems with proof

Objective

Apply the standard circle theorems with proof-style reasons, and use geometric reasoning to justify results (rows G8–G9, Extension).

1 Key knowledge and worked examples

The theorem bank. Centre angle = 2 × circumference angle (same arc); semicircle angle 90°; same segment, equal angles; cyclic quadrilateral opposite angles 180°; tangent perpendicular to the radius; tangents from an external point equal; perpendicular from centre bisects the chord; alternate segment theorem.

The verified chain. Circumference angle 35° → centre angle 70° (same arc) → the angle from the opposite arc is $\frac{1}{2}(360° - 70°) = 145°$, and $35° + 145° = 180°$ (cyclic quadrilateral): every step names its theorem, in full words.

2 Practice

2.1 An angle at the circumference is 41°. Write down the angle at the centre on the same arc. [1]

2.2 PQ is a diameter and R is on the circle. Find $\angle PRQ$. [1]

2.3 Two tangents from an external point T touch the circle at A and B. State the relationship between TA and TB. [1]

2.4 The perpendicular from the centre O to a chord XY meets it at M with $XM = 4.3$ cm. Write down MY. [1]

3 Exam-style questions

3.1 *(Original, 1E 35°/100° family.)* A, B, C, D lie on a circle centre O with $\angle ADB = 35°$ (D on the major arc AB). (a) Write down $\angle AOB$. (b) C lies on the minor arc AB: write down $\angle ACB$, with reasons. [4]

3.2 *(Original.)* TP and TQ are tangents to a circle centre O at P and Q; $\angle PTQ = 46°$. (a) Explain why $\angle OPT = 90°$. (b) Find $\angle POQ$. (c) Hence find $\angle PRQ$ for R on the major arc PQ. [4]

3.3 *(Original, alternate segment theorem.)* The tangent at A to a circle meets a line through B (also on the circle) at T, and C is on the circle with $\angle TAB = 62°$. Write down $\angle ACB$, with a reason. [2]

Solutions

2.1 $82°$.

2.2 $90°$ (angle in a semicircle).

2.3 $TA = TB$ (tangents from an external point are equal).

2.4 4.3 cm (the perpendicular from the centre bisects the chord).

3.1 (a) $\angle AOB = 2 \times 35° = 70°$ (centre angle is twice the circumference angle on the same arc). (b) $\angle ACB$ subtends the same chord from the OTHER arc, so it is half the reflex angle at the centre: $\frac{1}{2}(360° - 70°) = 145°$ – the two circumference angles are opposite angles of a cyclic quadrilateral summing to $180°$. ✓

3.2 (a) The tangent at any point is perpendicular to the radius at that point. (b) The quadrilateral $OPTQ$ has angles $90° + 90° + 46° + \angle POQ = 360°$, so $\angle POQ = 134°$. (c) $\angle PRQ = \frac{1}{2} \times 134° = 67°$ (centre is double the circumference on the same arc, R on the major arc).

3.3 $\angle ACB = 62°$ – the alternate segment theorem: the angle between the tangent and the chord equals the angle in the alternate segment.

Local review guidance: the theorem named in full words at every step; exact angles; cyclic-quadrilateral and reflex-arc cases justified. This is authored classroom guidance.