3.1 Angles, polygons, congruence and similarity

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS congruent • similar

Angles, polygons, congruence and similarity

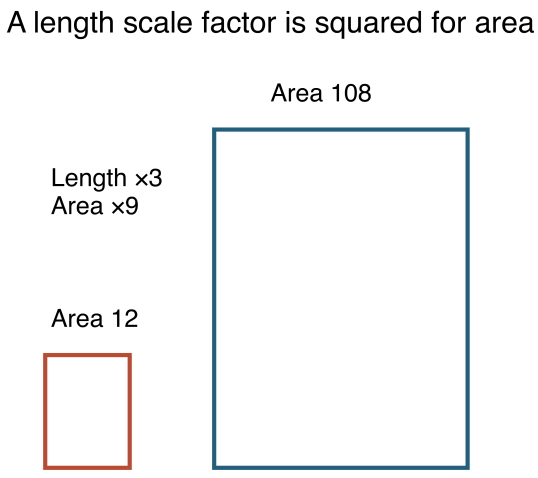
Objective

Use angle facts, parallel-line angles and polygon sums; quadrilateral properties and symmetry; congruence conditions and similarity ratios (rows G1–G7, G17 with the Extension additions).

1 Key knowledge and worked examples

Angles. Point 360°, line 180°, vertically opposite equal; alternate, corresponding and interior angles between parallels; triangle sum 180°; polygon interior $(n-2)\times 180°$, exterior $\frac{360°}{n}$, centre angle $\frac{360°}{n}$ for regular polygons.

Congruence and similarity. Conditions: SSS, SAS, ASA, RHS (Extension). Similar figures: lengths k, areas k^2, volumes k^3 – an area ratio $4:9$ means lengths $2:3$; the 2E specimen's similar solids give 864 cm^3 from a volume ratio.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 A regular polygon has interior angle 150°. Find n and the angle at the centre. [3]

2.2 Two angles of a triangle are 47° and 71°. Find the third and the exterior angle at that vertex. [2]

2.3 Write down the congruence condition for two triangles with two sides and the included angle equal. [1]

2.4 Similar jugs have heights 8 cm and 12 cm. The smaller holds 144 cm^3. Find the larger's volume. [3]

2.5 Similar photographs have areas 18 cm^2 and 50 cm^2. Find the ratio of their widths. [3]

3 Exam-style questions

3.1 *(Original, 2E SAS family.)* Triangles ABC and PQR have $AB = PQ = 7$ cm, $BC = QR = 9$ cm and $\angle ABC = \angle PQR = 54°$. (a) Prove the triangles are congruent. (b) Write down the length AC. [3]

3.2 *(Original, 2E similar-solids family.)* Two similar statues have heights in the ratio $3 : 5$; the smaller displaces 216 cm^3 of water. (a) Find the volume of the larger. (b) The larger's base area is 150 cm^2; find the smaller's. [4]

3.3 *(Original.)* Two lines are cut by a transversal: the two interior angles on one side of the transversal measure 63° and 117°. Explain, with reasons, why the two lines are parallel. [3]

Solutions

2.1 Exterior $= 30°$, so $n = 12$; centre angle $= 30°$.

2.2 Third angle $= 62°$; exterior $= 47 + 71 = 118°$.

2.3 SAS.

2.4 $k = \frac{12}{8} = \frac{3}{2}$; volumes scale by $\frac{27}{8}$: $144 \times \frac{27}{8} = 486$ cm^3.

2.5 $\sqrt{18} : \sqrt{50} = 3\sqrt{2} : 5\sqrt{2} = 3 : 5$.

3.1 (a) Two pairs of sides and the included angle are equal: SAS, so the triangles are congruent. (b) $AC = PR$ (corresponding sides) – but PR is given as 9 cm only if the correspondence maps $BC \to QR$; from SAS with $AB \leftrightarrow PQ$ and $BC \leftrightarrow QR$, the third sides correspond: $AC = PR$. With the data given, the cosine rule gives $AC^2 = 49 + 81 - 2(7)(9)\cos 54° = 130 - 126 \times 0.588 = 130 - 74.1 = 55.9$, so $AC = 7.5$ cm – and PR equals the same value by the same working.

3.2 (a) $k = \frac{5}{3}$; volume ratio $\frac{125}{27}$: $216 \times \frac{125}{27} = 1000$ cm^3. (b) Area ratio $\frac{25}{9}$: smaller base $= 150 \times \frac{9}{25} = 54$ cm^2.

3.3 Co-interior angles summing to $180°$ $(63° + 117°)$ mean the two lines are parallel – the interior-angle property between parallels.

Local review guidance: the named fact before the number; exact values; congruence names the condition and the correspondence. This is authored classroom guidance.