2.6 Sequences

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS quadratic sequence

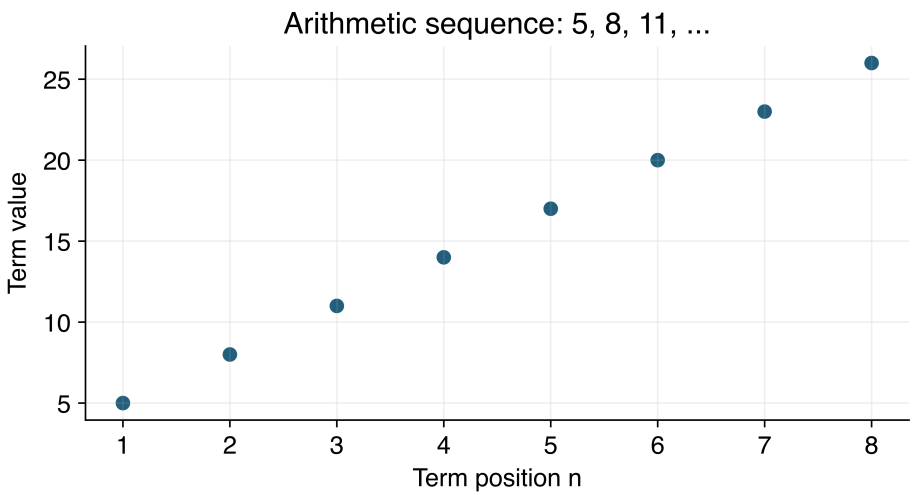
Sequences

Objective

Generate sequences from term-to-term and position-to-term rules; use triangular, square and cube sequences; find linear and quadratic nth terms (rows A24–A26 with the Extension quadratic-sequence addition).

1 Key knowledge and worked examples

nth terms. Arithmetic: $an + b$. Quadratic (Extension): second differences constant; $a = \frac{1}{2}$(second difference). For $5, 9, 17, 29, 45$: differences $4, 8, 12, 16$, second difference 4, so $a = 2$; subtracting $2n^2$ leaves $3, 1, -1, -3, -5 = -2n + 5$: nth term $2n^2 - 2n + 5$.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 Write the next two terms of $5, 9, 17, 29, 45, \ldots$ [2]

2.2 Find the nth term of $4, 7, 12, 19, 28, \ldots$ [3]

2.3 Find the nth term of $6, 11, 18, 27, 38, \ldots$ [3]

2.4 Which square number follows 144? [1]

3 Exam-style questions

3.1 *(Original, quadratic-sequence family.)* A sequence has nth term $3n^2 - 2n$. (a) Write down the first four terms. (b) Which term of the sequence equals 133? [4]

3.2 *(Original.)* The first differences of a sequence are $5, 9, 13, 17, \ldots$. (a) Find the next two differences. (b) Given the first term is 3, write the first four terms. (c) Find the nth term. [5]

3.3 *(Original, specification line not sampled: variation.)* y varies directly as the square of x; $y = 48$ when $x = 4$. (a) Find the equation. (b) Find y when $x = 7$. [4]

Solutions

2.1 Differences are $4, 8, 12, 16$, next differences $20, 24$: terms $65, 89$.

2.2 Differences $3, 5, 7, 9$; second difference 2, so $a = 1$: n^2 leaves $3, 3, 3, 3$ – nth term $n^2 + 3$.

2.3 Differences $5, 7, 9, 11$; second difference 2, so $a = 1$. Subtracting n^2 (i.e. 1, 4, 9, 16) leaves $5, 7, 9, 11 = 2n + 3$: nth term $n^2 + 2n + 3$.

2.4 169.

3.1 (a) $n = 1$: 1; $n = 2$: 8; $n = 3$: 21; $n = 4$: 40. (b) $3n^2 - 2n = 133$ gives $3n^2 - 2n - 133 = 0$, and $(3n + 19)(n - 7) = 0$, so $n = 7$ (reject $-\frac{19}{3}$). Check: $3(49) - 14 = 133$. ✓

3.2 (a) The differences themselves increase by 4: next differences 21 and 25. (b) Terms: $3,\ 3 + 5 = 8,\ 8 + 9 = 17,\ 17 + 13 = 30$. (c) Second difference of the sequence is 4, so $a = 2$; subtracting $2n^2$ leaves $1, 0, -1, -2 = -n + 2$: nth term $2n^2 - n + 2$ (check $n = 4$: $32 - 4 + 2 = 30$).

3.3 (a) $y = kx^2$: $48 = 16k$ gives $k = 3$, so $y = 3x^2$. (b) $x = 7$: $y = 3 \times 49 = 147$.

Local review guidance: second differences before guessing a; nth terms checked on two positions; variation parts state the model form. This is authored classroom guidance.