2.5 Equations, simultaneity and inequalities

Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS quadratic inequality

Equations, simultaneity and inequalities

Objective

Solve linear equations and quadratics by all three methods, linear pairs and linear–quadratic simultaneous pairs, and translate contexts into algebra (rows A18–A23 with the Extension additions).

1 Key knowledge and worked examples

Solving. Quadratics: factorise, complete the square, or the formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$. Simultaneity: substitute the linear into the quadratic; the 1E specimen's $x = -3 \pm \sqrt{7}$ comes from $x^2 + 6x + 2 = 0$.

Pairs and contexts. Two linear equations by elimination; one linear with one quadratic by substitution – keep BOTH points: $y = 2x + 1$ with $y = x^2 - 2$ meets at $(3, 7)$ and $(-1, -1)$. Contexts translate into the same equations with units.

2 Practice

2.1 Solve $5x - 3 = 2x + 9$ [2]

2.2 Solve $x^2 - 3x - 10 = 0$ [2]

2.3 Solve $x^2 + 6x + 2 = 0$, giving exact roots [3]

2.4 Solve the pair $y = 2x + 1$, $y = x^2 - 2$ [4]

2.5 Solve $\frac{x}{3} + 2 > \frac{x}{2}$ [3]

3 Exam-style questions

3.1 *(Original, 1E intersection family.)* A line $y = 2x + 5$ meets the curve $y = x^2 + 1$ at two points. (a) Show the x-coordinates satisfy $x^2 - 2x - 4 = 0$. (b) Find both intersection points, exact where possible. [5]

3.2 *(Original.)* A rectangular field's length exceeds its width by 5 m; its area is 84 m^2. Form and solve a quadratic to find both dimensions. [4]

3.3 *(Original, 2E inequality family.)* Solve and represent on a number line: $-7 \leq 3x+2 < 6$. [3]

Solutions

2.1 $3x = 12$, so $x = 4$.

2.2 $(x-5)(x+2) = 0$: $x = 5$ or -2.

2.3 $x = \dfrac{-6 \pm \sqrt{36-8}}{2} = \dfrac{-6 \pm \sqrt{28}}{2} = -3 \pm \sqrt{7}$.

2.4 $x^2 - 2 = 2x + 1$, so $x^2 - 2x - 3 = 0 = (x-3)(x+1)$: $x = 3$ or -1, giving $(3, 7)$ and $(-1, -1)$.

2.5 Multiply by 6: $2x + 12 > 3x$, so $x < 12$.

3.1 (a) $2x + 5 = x^2 + 1$ gives $x^2 - 2x - 4 = 0$. (b) $x = \dfrac{2 \pm \sqrt{4+16}}{2} = 1 \pm \sqrt{5}$; points $\left(1+\sqrt{5},\, 7+2\sqrt{5}\right)$ and $\left(1-\sqrt{5},\, 7-2\sqrt{5}\right)$.

3.2 Width w: $w(w+5) = 84$, so $w^2 + 5w - 84 = 0 = (w+12)(w-7)$: width 7 m, length 12 m (reject -12).

3.3 Subtract 2 throughout: $-9 \le 3x < 4$; divide by 3: $-3 \le x < \frac{4}{3}$; closed circle at -3, open at $\frac{4}{3}$.

Local review guidance: substitution shown before expansion; exact roots and both intersection points; context rejections justified. This is authored classroom guidance.