2.4 Differentiation

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS tangent • maxima or minima

Differentiation

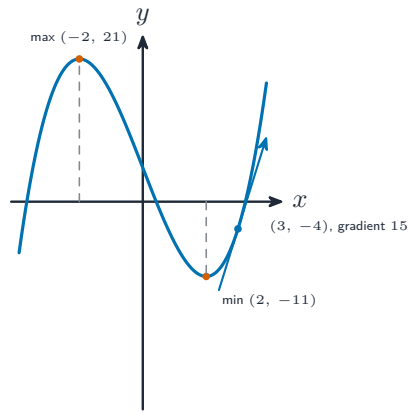
Objective

Differentiate kx^n and sums, use the gradient function, find tangent equations and stationary points classified as maxima, minima or inflections (rows A13–A15, Extension).

1 Key knowledge and worked examples

The gradient function. $\frac{d}{dx}(kx^n) = knx^{n-1}$ term by term; the gradient at a point equals the tangent's gradient there. For $y = x^2$: at $x = 2$, $m = 4$ and the tangent is $y = 4x - 4$.

Stationary points. Solve $\frac{dy}{dx} = 0$; classify by the sign change (or the sketch shape). For $y = x^3 - 12x + 5$: $3x^2 - 12 = 0$ gives $x = \pm 2$ – maximum $(-2, 21)$, minimum $(2, -11)$; at $x = 3$ the gradient is 15 and the tangent is $y = 15x - 49$ (the 2E specimen's dy/dx = 15 family).



tangent (blue) $y = 15x - 49$;
stationary where $3x^2 - 12 = 0$

2 Practice

2.1 Differentiate $y = x^3 - 12x + 5$ [2]

2.2 Differentiate $y = 4x^5 + 3x^2 - 7$ [2]

2.3 Find the gradient of $y = x^2$ at $x = 2$ and the tangent's equation [3]

2.4 Find the stationary points of $y = x^3 - 12x + 5$ [3]

3 Exam-style questions

3.1 *(Original, 2E tangent family.)* The curve $y = x^3 - 12x + 5$ has $\dfrac{dy}{dx} = 3x^2 - 12$. (a) Verify that the gradient at $x = 3$ is 15. (b) Find the equation of the tangent at $(3, -4)$. [4]

3.2 *(Original.)* A curve has $y = 2x^3 + 9x^2 - 24x + 1$. (a) Find $\dfrac{dy}{dx}$. (b) Find both stationary points' coordinates. (c) Classify each from the sketch shape. [6]

3.3 *(Original, specification line not sampled: point of inflection.)* For $y = x^3$, explain why $x = 0$ is a stationary point but neither a maximum nor a minimum. [3]

Solutions

2.1 $3x^2 - 12$.

2.2 $20x^4 + 6x$.

2.3 $\dfrac{dy}{dx} = 2x$, so $m = 4$; $y - 4 = 4(x - 2)$ gives $y = 4x - 4$.

2.4 $x = \pm 2$: maximum $(-2, 21)$, minimum $(2, -11)$.

3.1 (a) $3(3)^2 - 12 = 27 - 12 = 15$. ✓ (b) $y + 4 = 15(x - 3)$, so $y = 15x - 49$.

3.2 (a) $6x^2 + 18x - 24$. (b) $6x^2 + 18x - 24 = 0$ gives $x^2 + 3x - 4 = 0$, so $(x+4)(x-1) = 0$: $x = -4$ and $x = 1$. Points: at $x = -4$, $y = -128 + 144 + 96 + 1 = 113$; at $x = 1$, $y = 2 + 9 - 24 + 1 = -12$. (c) Positive cubic: maximum $(-4, 113)$, minimum $(1, -12)$.

3.3 $\dfrac{dy}{dx} = 3x^2 = 0$ at $x = 0$ (stationary), but the gradient is positive on BOTH sides of $x = 0$ – the curve keeps rising, so the point is a point of inflection, not a turning point.

Local review guidance: differentiation before any substitution; exact coordinates; classification argues from gradient signs or shape. This is authored classroom guidance.