

2.3 Lines, curves and inequalities

Name: _____ Class: _____ Date: _____

Total: 25 marks

KEY WORDS completing the square

Lines, curves and inequalities

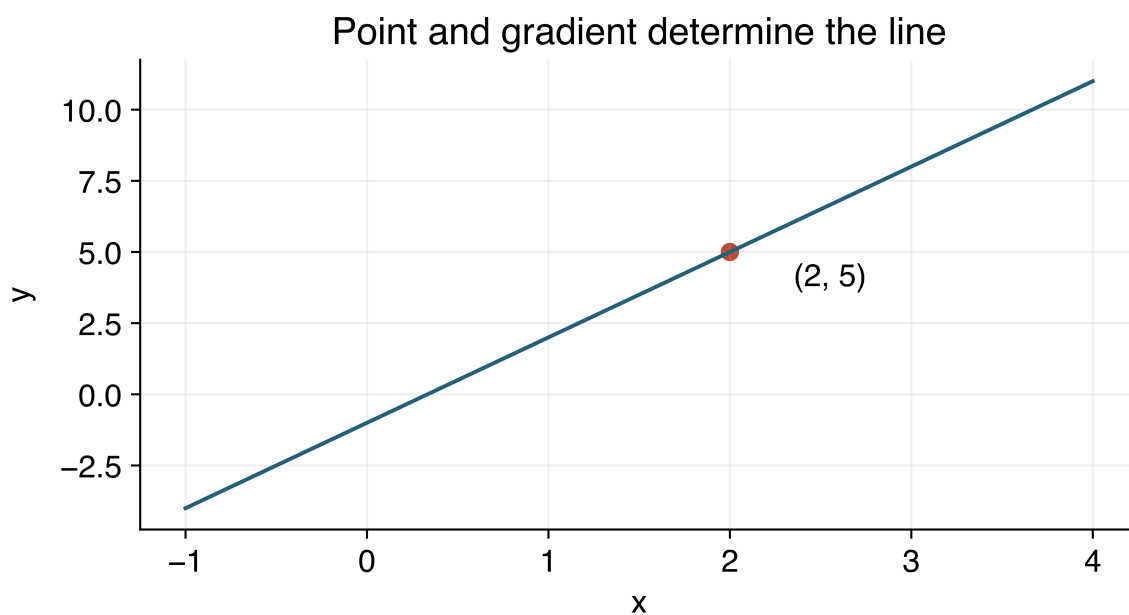
Objective

Find line equations through two points or a point with a gradient; use perpendicular gradients; sketch curve families; solve linear and quadratic inequalities (rows A10–A12, A16, A23 with the Extension additions; several lines unsampled).

1 Key knowledge and worked examples

Lines. $m = \frac{y_2 - y_1}{x_2 - x_1}$; through a point: $y - y_1 = m(x - x_1)$. Through (1, 7) and (2, 9): $m = 2$, $y = 2x + 5$. Perpendicular gradients (not sampled): $m_1 m_2 = -1$.

Curves and inequalities. Sketch linear, quadratic, cubic, reciprocal graphs; exponential $y = k^x$ and trig graphs (not sampled). Turning points by completing the square: $x^2 - 6x + 5 = (x - 3)^2 - 4$, minimum (3, -4) (not sampled). Quadratic inequality: $x^2 - x - 6 > 0 \Rightarrow x < -2$ or $x > 3$.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 Find the equation of the line through (1, 7) and (2, 9) [3]

2.2 Find the equation of the line through (0, -3) with gradient 4 [2]

2.3 Write down the gradient perpendicular to $m = \frac{2}{5}$ [1]

2.4 Solve $x^2 - x - 6 > 0$ [3]

2.5 Solve $-7 \leq 3x + 2 < 6$ (the 2E specimen's answer family)

[3]

3 Exam-style questions

3.1 (*Original.*) A line joins $A(-1, 3)$ to $B(3, 11)$. (a) Find its equation. (b) Find the equation of the perpendicular line through the midpoint of AB .

[5]

3.2 (*Original, specification line not sampled: completing the square.*) Write $x^2 + 6x + 2$ in the form $(x + a)^2 + b$, hence state the minimum point and solve $x^2 + 6x + 2 = 0$ giving exact roots (the 1E specimen's answer family).

[5]

3.3 (*Original.*) Sketch or describe the graph of $y = 2^x$, giving the y -intercept and the behaviour as x grows and as $x \rightarrow -\infty$.

[3]

Solutions

2.1 $m = 2$, so $y = 2x + 5$.

2.2 $y = 4x - 3$.

2.3 $-\frac{5}{2}$.

2.4 $(x - 3)(x + 2) > 0$: positive outside the roots, so $x < -2$ or $x > 3$.

2.5 Subtract 2 throughout: $-9 \leq 3x < 4$; divide by 3: $-3 \leq x < \frac{4}{3}$.

3.1 (a) $m = \frac{11-3}{3-(-1)} = 2$, so $y = 2x + 5$. (b) Midpoint $(1, 7)$; perpendicular gradient $-\frac{1}{2}$: $y - 7 = -\frac{1}{2}(x - 1)$, so $y = -\frac{1}{2}x + \frac{15}{2}$.

3.2 $x^2 + 6x + 2 = (x + 3)^2 - 7$. Minimum $(-3, -7)$; roots: $(x + 3)^2 = 7$, so $x = -3 \pm \sqrt{7}$.

3.3 Passes through $(0, 1)$; rises ever faster as x grows; as $x \rightarrow -\infty$ the curve approaches the x -axis (never reaches 0).

Local review guidance: gradient then point-slope form; exact roots; inequality solution sets stated with the correct strict/non-strict signs. This is authored classroom guidance.