2.2 Functions and proof

Name: _____ Class: _____ Date: _____ **Total: 19 marks**

KEY WORDS domain • proof

Functions and proof

Objective

Use function notation with domain and range, build composites and inverses, and construct formal algebraic proofs (rows A8–A9 with the Extension additions).

1 Key knowledge and worked examples

Functions. $\mathrm{f}(x)$ maps each input to one output; domain is the input set, range the output set. Composite $\mathrm{fg}(x) = \mathrm{f}(g(x))$ – apply g first. Inverse by swap-and-rearrange; the 1E specimen has $\mathrm{f}^{-1}(x) = \frac{x}{3}$ from $\mathrm{f}(x) = 3x$.

Proof. State the algebra, simplify, compare with the claim: $(2n+1)^2-(2n-1)^2 = 8n$ – a multiple of 8 for every integer n. The 2E specimen proves the $n^2 + n + 9$ family. Examples never prove.

2 Practice

2.1 For $\mathrm{f}(x) = 3x - 5$, find $\mathrm{f}(4)$ [1]

2.2 For $\mathrm{f}(x) = 3x - 5$ and $\mathrm{g}(x) = x^2$, find $\mathrm{fg}(2)$ [2]

2.3 Find $\mathrm{f}^{-1}(x)$ for $\mathrm{f}(x) = 3x$ [2]

2.4 State the domain and range of $\mathrm{f}(x) = \sqrt{x-2}$ [2]

3 Exam-style questions

3.1 *(Original, 2E composite family.)* $\mathrm{g}(x) = x - 5$ and $\mathrm{f}(x) = x^2 + 1$. (a) Find $\mathrm{fg}(x)$. (b) Solve $\mathrm{fg}(x) = 5$. [4]

3.2 *(Original, 1E inverse family.)* $\mathrm{f}(x) = \dfrac{x}{3} + 2$. (a) Find $\mathrm{f}^{-1}(x)$. (b) Verify that $\mathrm{f}^{-1}(14) = 36$. [4]

3.3 *(Original, 2E proof family.)* Prove that the difference of the squares of any two consecutive odd numbers is always a multiple of 8. [4]

Solutions

2.1 7.

2.2 $\mathrm{g}(2) = 4$, then $\mathrm{f}(4) = 7$.

2.3 $\mathrm{f}^{-1}(x) = \frac{x}{3}$.

2.4 Domain $x \geq 2$; range $\mathrm{f}(x) \geq 0$.

3.1 (a) $\mathrm{fg}(x) = (x-5)^2 + 1$. (b) $(x-5)^2 = 4$, so $x = 3$ or $x = 7$.

3.2 (a) $y = \frac{x}{3} + 2 \Rightarrow x = 3(y-2)$, so $\mathrm{f}^{-1}(x) = 3x - 6$. (b) $\mathrm{f}^{-1}(14) = 42 - 6 = 36$. ✓

3.3 Consecutive odd numbers: $2n+1$ and $2n-1$. Their squares differ by $(2n+1)^2 - (2n-1)^2 = (4n^2+4n+1) - (4n^2-4n+1) = 8n$, a multiple of 8 for every integer n.

Local review guidance: composite order – g first inside f; inverses with domains; proofs end with the conclusion in words. This is authored classroom guidance.