1.4 Ratio, rate, proportion and exponentials

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS exponential

Ratio, rate, proportion and exponentials

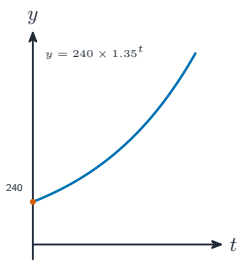
Objective

Simplify and share ratios, apply ratio to problems, use rates including pay, work with direct and inverse proportion and exponential growth/decay models (rows N16–N20 with the Extension exponential addition).

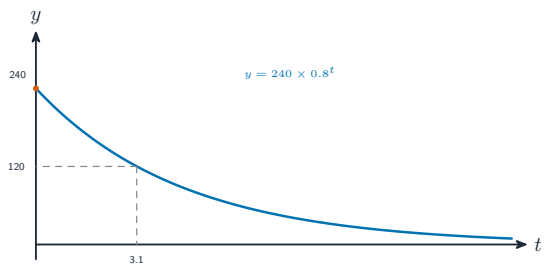
1 Key knowledge and worked examples

Ratio and rate. Simplify $45 : 60 = 3 : 4$; share £350 in $2 : 5$ (£100/£250). Rates: £9.20/h for 7.5 h = £69. Direct proportion $y = kx$; inverse $y = k/x$.

Exponentials. Growth $y = ab^t$, $b > 1$; decay $0 < b < 1$. From the data 240, 192, 153.6: ratio 0.8, so $y = 240 \times 0.8^t$ (2E specimen family); values halve every $0.8^t = \frac{1}{2}$, i.e. $t \approx 3.1$ years.



growth: the step itself grows each year



decay: multiplies by 0.8 each year, never reaching zero

2 Practice

2.1 Simplify $45 : 60$ [1]

2.2 Share £350 in the ratio $2 : 5$ [3]

2.3 A job pays £9.20 per hour. Find the pay for 7.5 hours. [2]

2.4 $y \propto x$ with $y = 21$ when $x = 7$: find the equation and y when $x = 12$ [3]

2.5 A sequence of values reads 240, 192, 153.6. Find the ratio and state the model $y = ab^t$. [3]

3 Exam-style questions

3.1 *(Original.)* A recipe for 6 people uses 450 g of rice. (a) How much for 10 people? (b) With 525 g available, how many whole people can be fed? [4]

3.2 *(Original, 2E specimen family.)* A monitor reading decays by 20% each year from 240 units; the model is $y = 240 \times 0.8^t$. (a) Find the half-life (when y first falls below 120). (b) A second reading GROWS by 35% per year from 240: write its model and find the value when $t = 5$. [5]

3.3 *(Original.)* y is inversely proportional to x; $y = 4$ when $x = 6$. Find y when $x = 8$. [3]

Solutions

2.1 $3:4$.

2.2 £100 and £250.

2.3 $9.2 \times 7.5 =$ £69.

2.4 $k = 3$, so $y = 3x$; $y = 36$.

2.5 Ratio $\frac{192}{240} = 0.8$ (and $\frac{153.6}{192} = 0.8$): $y = 240 \times 0.8^t$.

3.1 (a) $450 \times \frac{10}{6} = 750$ g. (b) Per person 75 g; $525 \div 75 = 7$ people.

3.2 (a) $240 \times 0.8^t < 120$ needs $0.8^t < 0.5$, so $t > \ln 0.5 / \ln 0.8 \approx 3.1$: first below 120 during year 4 (at $t = 4$, $y = 98.3$). (b) $y = 240 \times 1.35^t$; at $t = 5$: $240 \times 1.35^5 = 240 \times 4.484 = 1076.2$.

3.3 $y = \frac{k}{x}$: $k = 24$, so $y = 3$.

Local review guidance: the constant or ratio found from one data pair first; exact amounts; exponential answers name the model used. This is authored classroom guidance.