

1.4 Ratio, rate, proportion and exponentials

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS exponential

Ratio, rate, proportion and exponentials

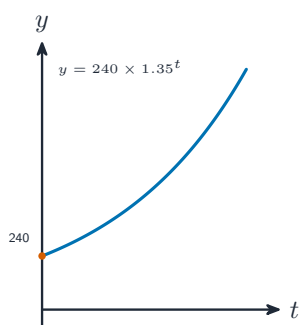
Objective

Simplify and share ratios, apply ratio to problems, use rates including pay, work with direct and inverse proportion and exponential growth/decay models (rows N16–N20 with the Extension exponential addition).

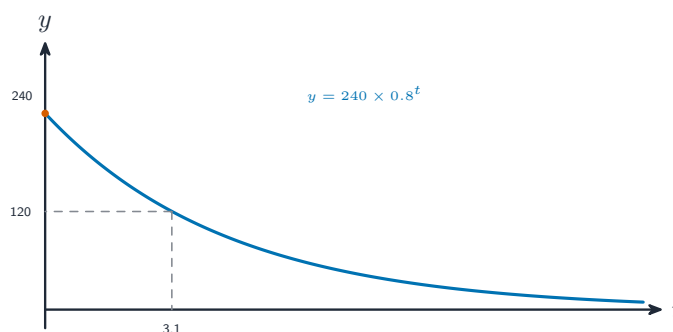
1 Key knowledge and worked examples

Ratio and rate. Simplify $45 : 60 = 3 : 4$; share £350 in $2 : 5$ (£100/£250). Rates: £9.20/h for 7.5 h = £69. Direct proportion $y = kx$; inverse $y = k/x$.

Exponentials. Growth $y = ab^t$, $b > 1$; decay $0 < b < 1$. From the data 240, 192, 153.6: ratio 0.8, so $y = 240 \times 0.8^t$ (2E specimen family); values halve every $0.8^t = \frac{1}{2}$, i.e. $t \approx 3.1$ years.



growth: the step
itself grows each year



decay: multiplies by 0.8 each
year, never reaching zero

2 Practice

2.1 Simplify 45 : 60 [1]

2.2 Share £350 in the ratio 2 : 5 [3]

2.3 A job pays £9.20 per hour. Find the pay for 7.5 hours. [2]

2.4 $y \propto x$ with $y = 21$ when $x = 7$: find the equation and y when $x = 12$ [3]

2.5 A sequence of values reads 240, 192, 153.6. Find the ratio and state the model $y = ab^t$. [3]

3 Exam-style questions

3.1 (*Original.*) A recipe for 6 people uses 450 g of rice. (a) How much for 10 people? (b) With 525 g available, how many whole people can be fed? [4]

3.2 (*Original, 2E specimen family.*) A monitor reading decays by 20% each year from 240 units; the model is $y = 240 \times 0.8^t$. (a) Find the half-life (when y first falls below 120). (b) A second reading GROWS by 35% per year from 240: write its model and find the value when $t = 5$. [5]

3.3 (*Original.*) y is inversely proportional to x ; $y = 4$ when $x = 6$. Find y when $x = 8$. [3]

Solutions

2.1 $3 : 4$.

2.2 £100 and £250.

2.3 $9.2 \times 7.5 = \text{£}69$.

2.4 $k = 3$, so $y = 3x$; $y = 36$.

2.5 Ratio $\frac{192}{240} = 0.8$ (and $\frac{153.6}{192} = 0.8$): $y = 240 \times 0.8^t$.

3.1 (a) $450 \times \frac{10}{6} = 750$ g. (b) Per person 75 g; $525 \div 75 = 7$ people.

3.2 (a) $240 \times 0.8^t < 120$ needs $0.8^t < 0.5$, so $t > \ln 0.5 / \ln 0.8 \approx 3.1$: first below 120 during year 4 (at $t = 4$, $y = 98.3$). (b) $y = 240 \times 1.35^t$; at $t = 5$: $240 \times 1.35^5 = 240 \times 4.484 = 1076.2$.

3.3 $y = \frac{k}{x}$: $k = 24$, so $y = 3$.

Local review guidance: the constant or ratio found from one data pair first; exact amounts; exponential answers name the model used. This is authored classroom guidance.