Exercise walk-through

# 4.3 Averages, quartiles and percentiles

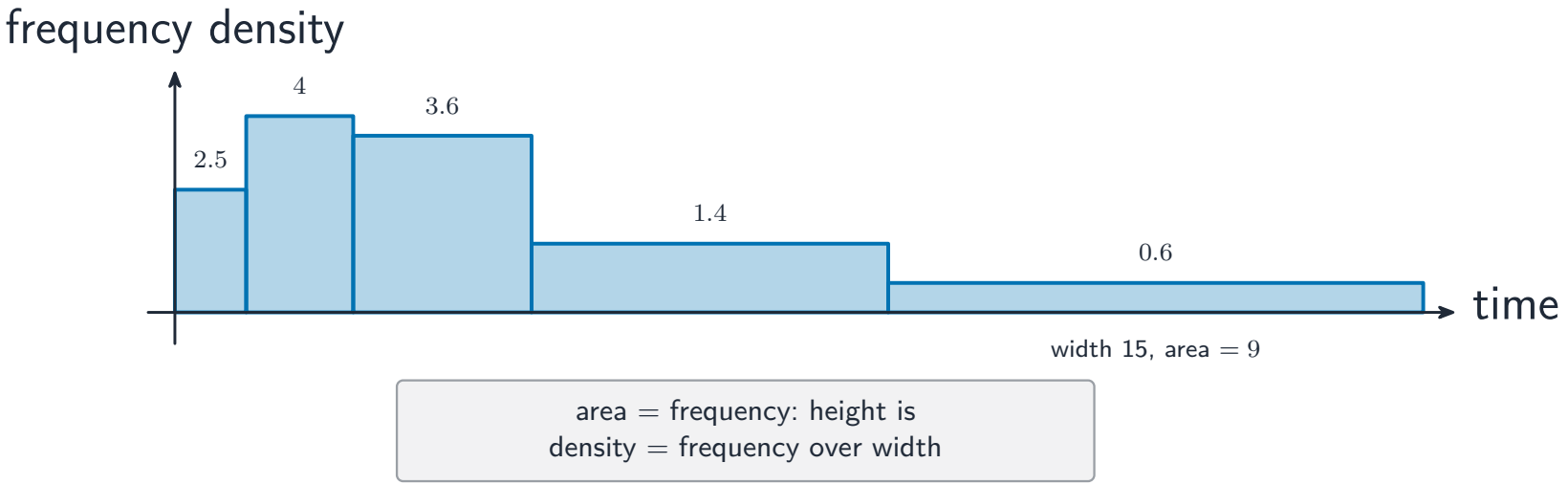
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# Extension diagrams: unequal histograms, CF and box plots

- (Extension) A **histogram** can have **unequal class intervals** .
- Then the height of each bar is the **frequency density** :
- The area of each bar is the frequency.
- For classes of widths 2, 3, 5, 10 and 15 with frequencies 5, 12, 18, 14 and 9, the heights are 2.5, 4, 3.6, 1.4 and 0.6.

$$\text{frequency density} = \frac{\text{frequency}}{\text{class width}}$$

# Figure: Extension diagrams: unequal histograms, CF and box plots




area = frequency: height is
density = frequency over width

- Height = **frequency density**
- Area of each bar = frequency

# Exercises 2.1·2.2

**2.1** Find the mean, the median and the mode of these numbers. [2]

2 4 4 6 9

**2.2** Find the lower quartile, the upper quartile and the interquartile range of these 11 numbers. [2]

3 5 6 8 9 11 12 14 15 18 20

# Exercises 2.1·2.2

**2.1** Find the mean, the median and the mode of these numbers. [2]

2 4 4 6 9

**2.1**

- mean $= \frac{25}{5} = \mathbf{5}$
- median **4**, mode **4**

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**2.2** Find the lower quartile, the upper quartile and the interquartile range of these 11 numbers. [2]

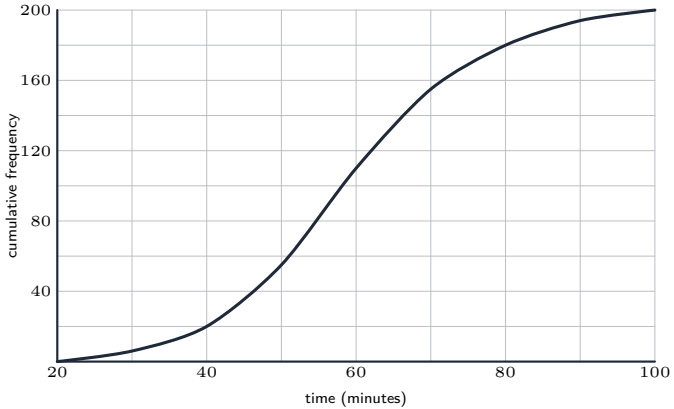
3 5 6 8 9 11 12 14 15 18 20

**2.2**

- $Q_1 = \mathbf{6}$ (3rd value), $Q_3 = \mathbf{15}$ (9th value)
- IQR $= \mathbf{9}$

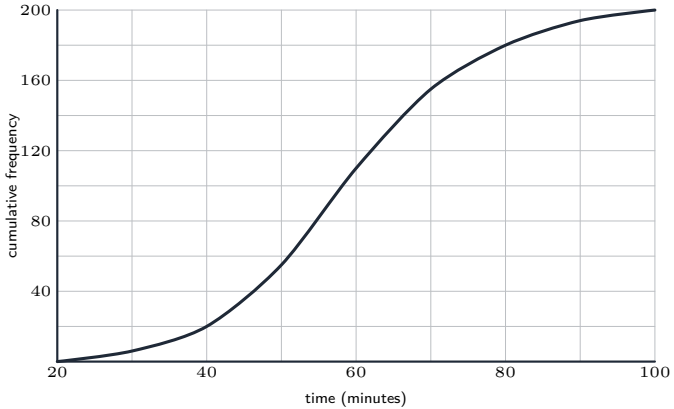
# Exercise 2.3

**2.3** The graph shows the times of 200 runners. At which cumulative frequency do you read the 90th percentile? Use the graph to find the difference between the 90th and the 10th percentiles. [2]



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**2.3**

- $\frac{90}{100} \times 200 = \mathbf{180}$
- $80 - 40 = \mathbf{40}$ minutes

# Exercise 3.1

**3.1** The table shows the number of books some students read in a month. The mean is 2 books. Find $k$. [3]

| books | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| frequency | 4 | 6 | 8 | $k$ | 2 |

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**3.1**

- $\sum fx = 0 + 6 + 16 + 3k + 8 = 30 + 3k$ and $\sum f = 20 + k$
- $30 + 3k = 2(20 + k)$
- $k = \mathbf{10}$

# Exercise 3.2 (a)

**3.2** The table shows the masses of 80 apples from a farm.

(a) Write down the modal class. Which class contains the median? [2]

| mass $m$ (g) | frequency |
|---|---|
| $100 < m \leq 120$ | 6 |
| $120 < m \leq 140$ | 18 |
| $140 < m \leq 160$ | 30 |
| $160 < m \leq 180$ | 20 |
| $180 < m \leq 200$ | 6 |

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| $180 < m \leq 200$ | 6 |

(a) modal class $\mathbf{140 < m \leq 160}$; cumulative frequencies 6, 24, 54, so the median is in $\mathbf{140 < m \leq 160}$

# Exercise 3.2 (b)

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(b) midpoints 110, 130, 150, 170, 190; $\frac{660+2340+4500+3400+1140}{80} = \frac{12040}{80} = \mathbf{150.5}$ g

# Exercise 3.2 (c)

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(c) read at the 72nd and the 20th values; $160 + \frac{18}{20} \times 20$ or $120 + \frac{14}{18} \times 20$; 90th percentile $=$ **178** g; $Q_1 =$ **136** g

# Exercise 3.2 (d)

**3.2** The table shows the masses of 80 apples from a farm.

(d) Apples heavier than 175 g are sold as “large”. Estimate the percentage of large apples. A shop buys the crop at a high price only if the 90th percentile is at least 180 g. Does this crop get the high price? Give a reason. [5]

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(d) 175 to 180: $\frac{5}{20} \times 20 = 5$; $5 + 6 = 11$; $\frac{11}{80} = \mathbf{13.75}\%$; the 90th percentile is 178 g, less than 180 g; so **no**, the crop does not get the high price