Exercise walk-through

3.4
Trigonometry in 2D and 3D

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Angles, polygons and symmetry

- Angles at a point add up to $360°$; angles on a straight line add up to $180°$.
- **Vertically opposite** angles are equal.
- With parallel lines, **alternate angles** are equal, **corresponding angles** are equal, and co-interior angles add up to $180°$. Write these names, never "Z angles" or "F angles".
- The angles in a triangle add up to $180°$.

Pythagoras, trigonometry and 3D problems

- $c^2 = 9^2 + 11^2 - 2 \times 9 \times 11 \times \cos 72° = 202 - 61.2 = 140.8$, so $c = 11.9$ (3 s.f.).
- $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{121 + 140.8 - 81}{2 \times 11 \times 11.87} = 0.693$, so $A = 46.2°$.
- Check: $B = 180° - 72° - 46.2° = 61.8°$, and the smallest angle A is opposite the shortest side a.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \quad \cos\theta = \frac{\text{adj}}{\text{hyp}}, \quad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

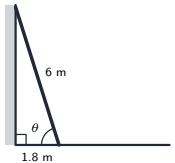
Transformations with matrices and vectors

- The first transformation goes on the right:
  $$\mathbf{RM} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.$$
- This is a reflection in the y-axis.
- Check with $(2, 1)$: the reflection gives $(1, 2)$, the rotation gives $(-2, 1)$, and a reflection in the y-axis also gives $(-2, 1)$.

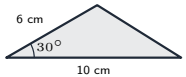
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

Exercises 2.1·2.2

2.1 A ladder of length 6 m leans against a wall. The foot of the ladder is 1.8 m from the wall. Find the angle between the ladder and the ground. [2]

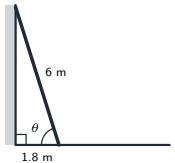


2.2 Find the area of the triangle. [2]



Exercises 2.1·2.2

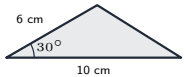
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2.1

- $\cos\theta = \frac{1.8}{6} = 0.3$
- $\theta = \mathbf{72.5}^\circ$

2.2 Find the area of the triangle. [2]



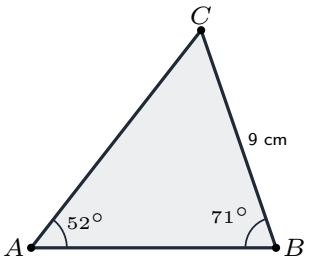
2.2

- $\frac{1}{2} \times 6 \times 10 \times \sin 30^\circ$
- $= \mathbf{15}\ \text{cm}^2$

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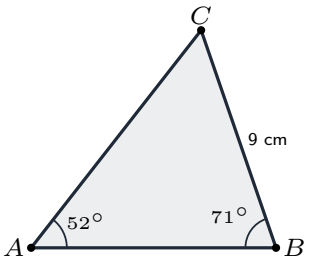
Exercise 2.3

2.3 In triangle ABC, angle $A = 52°$, angle $B = 71°$ and $BC = 9$ cm. Find the length of AC. [2]



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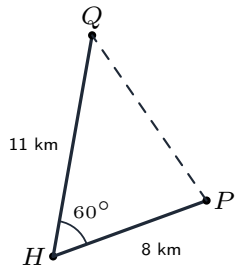


2.3

- $\frac{AC}{\sin 71°} = \frac{9}{\sin 52°}$
- $AC = \mathbf{10.8}$ cm

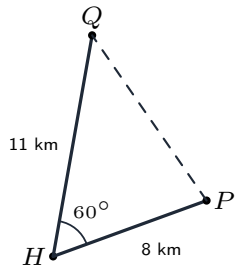
Exercise 3.1

3.1 Two boats leave a harbour H. One sails 8 km to P. The other sails 11 km to Q. The angle PHQ is 60°. Find the distance PQ. [3]



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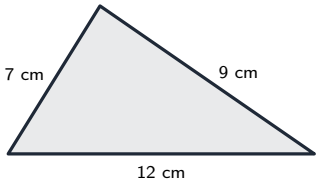


3.1

- $PQ^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \cos 60°$
- $= 64 + 121 - 88 = 97$
- $PQ = \mathbf{9.85}$ km

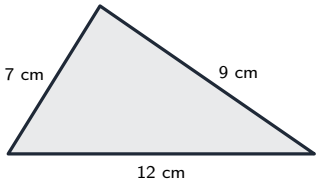
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3.2 A triangle has sides 7 cm, 9 cm and 12 cm. Find the largest angle of the triangle. [3]



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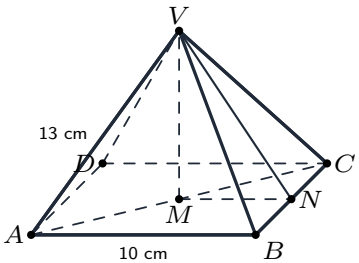
3.2

- the largest angle is opposite the 12 cm side
- $\cos\theta = \frac{49+81-144}{2\times7\times9} = -\frac{14}{126}$
- $\theta = \mathbf{96.4}^\circ$

Exercise 3.3 (a)

3.3 $VABCD$ is a pyramid with a square base $ABCD$ of side 10 cm. The top V is directly above the centre M of the base. Each sloping edge, such as VA, is 13 cm. N is the midpoint of BC.

(a) Find the height VM of the pyramid. [3]

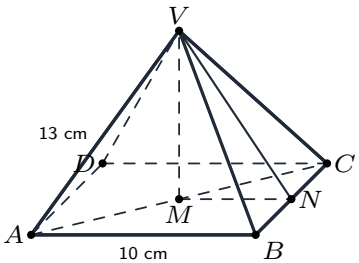


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(a) Find the height VM of the pyramid. [3]



(a) $AC = \sqrt{10^2 + 10^2} = 14.14$, so $AM = 7.071$; $VM^2 = 13^2 - 7.071^2 = 119$; $VM =$ **10.9** cm

Exercise 3.3 (b)

3.3 $VABCD$ is a pyramid with a square base $ABCD$ of side 10 cm. The top V is directly above the centre M of the base. Each sloping edge, such as VA, is 13 cm. N is the midpoint of BC.

(b) Find the angle between the edge VA and the base. [3]

Exercise 3.3 (b)

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(b) Find the angle between the edge VA and the base. [3]

(b) the angle is VAM; $\cos VAM = \frac{7.071}{13}$; $\mathbf{57.0}°$

Exercise 3.3 (c)

3.3 $VABCD$ is a pyramid with a square base $ABCD$ of side 10 cm. The top V is directly above the centre M of the base. Each sloping edge, such as VA, is 13 cm. N is the midpoint of BC.

(c) Find the angle between the face VBC and the base. [4]

Exercise 3.3 (c)

3.3 $VABCD$ is a pyramid with a square base $ABCD$ of side 10 cm. The top V is directly above the centre M of the base. Each sloping edge, such as VA, is 13 cm. N is the midpoint of BC.

(c) Find the angle between the face VBC and the base. [4]

(c) the angle is VNM, with $MN = 5$ cm; triangle VMN has a right angle at M; $\tan VNM = \frac{10.91}{5}$; $\mathbf{65.4}^\circ$

Exercise 3.3 (d)

3.3 $VABCD$ is a pyramid with a square base $ABCD$ of side 10 cm. The top V is directly above the centre M of the base. Each sloping edge, such as VA, is 13 cm. N is the midpoint of BC.

(d) Find angle AVB. Use $\frac{1}{2}ab\sin C$ to find the area of triangle VAB. Then find the total surface area of the pyramid. [5]

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(d) $\cos AVB = \frac{13^2+13^2-10^2}{2\times13\times13} = \frac{238}{338}$; $AVB = 45.2°$; area $= \frac{1}{2} \times 13 \times 13 \times \sin 45.2° = 60.0$ cm^2; total $= 4 \times 60 + 10^2 = \mathbf{340}$ cm^2