Exercise walk-through

3.3
Mensuration, sectors and solids

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Angles, polygons and symmetry

- Angles at a point add up to $360°$; angles on a straight line add up to $180°$.
- **Vertically opposite** angles are equal.
- With parallel lines, **alternate angles** are equal, **corresponding angles** are equal, and co-interior angles add up to $180°$. Write these names, never "Z angles" or "F angles".
- The angles in a triangle add up to $180°$.

Pythagoras, trigonometry and 3D problems

- $c^2 = 9^2 + 11^2 - 2 \times 9 \times 11 \times \cos 72° = 202 - 61.2 = 140.8$, so $c = 11.9$ (3 s.f.).
- $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{121 + 140.8 - 81}{2 \times 11 \times 11.87} = 0.693$, so $A = 46.2°$.
- Check: $B = 180° - 72° - 46.2° = 61.8°$, and the smallest angle A is opposite the shortest side a.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \quad \cos\theta = \frac{\text{adj}}{\text{hyp}}, \quad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Transformations with matrices and vectors

- The first transformation goes on the right:
  $$\mathbf{RM} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.$$
- This is a reflection in the y-axis.
- Check with $(2, 1)$: the reflection gives $(1, 2)$, the rotation gives $(-2, 1)$, and a reflection in the y-axis also gives $(-2, 1)$.

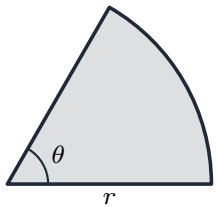
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

Exercises 2.1·2.2

2.1 Find the area of a trapezium with parallel sides 7 and 11 and height 4 [2]



2.2 A sector has $r = 9$, $\theta = 40°$. Find its arc and area. [3]



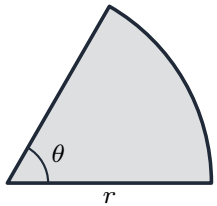
Exercises 2.1–2.2

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2.1 $\frac{1}{2}(7+11)\times 4 = 36$.

2.2 A sector has $r = 9$, $\theta = 40°$. Find its arc and area. [3]



2.2 Arc $= \frac{40}{360}\times 18\pi = 2\pi$; area $= \frac{40}{360}\times 81\pi = 9\pi$.

Exercise 2.3

2.3 Find the volume and surface area of a sphere of radius 6, in terms of π. [3]



2.4 A cone has $r = 5$ and $h = 12$. Find its slant, volume and curved surface, in terms of π. [4]



2.5 A cylinder has $r = 4$, $h = 10$. Find its volume and total surface area, in terms of π. [3]



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2.4 $l = \sqrt{25 + 144} = 13$; $V = \frac{1}{3}\pi(25)(12) = 100\pi$; curved $= \pi(5)(13) = 65\pi$.

2.5 $V = \pi(16)(10) = 160\pi$; $SA = 2\pi(16) + 2\pi(4)(10) = 32\pi + 80\pi = 112\pi$.

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Full solution 2.3

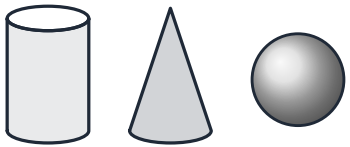
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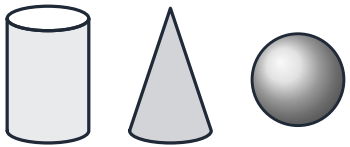
Exercise 3.1

3.1 *(Original, 1E cone-proportion family.)* A frustum is made by removing a cone of radius $3r$ and height $4r$ from a similar cone of radius $5r$ and height $\frac{20}{3}r$. (a) Show that the removed volume is $12\pi r^3$. (b) Show that the frustum's volume is $\frac{392}{9}\pi r^3$. (c) The large cone's radius is 15 cm; find the removed cone's height. [5]



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3.1 (a) Removed cone: $\frac{1}{3}\pi(3r)^2(4r) = \frac{1}{3}\pi\times 36r^3 = 12\pi r^3$. ✓ (b) Large cone: $\frac{1}{3}\pi(5r)^2\left(\frac{20}{3}r\right) = \frac{500}{9}\pi r^3$; frustum $= \frac{500}{9}\pi - \frac{108}{9}\pi = \frac{392}{9}\pi r^3$. ✓ (c) Radius $15 = 5$r gives $r = 3$; the large height is $\frac{20}{3}\times 3 = 20$ cm, and similar cones have height ratio $\frac{3}{5}$, so the removed height is $\frac{3}{5}\times 20 = 4r = 12$ cm.

Exercises 3.2·3.3

3.2 *(Original.)* A silo is a cylinder ($r = 3$ m, $h = 7$ m) topped by a hemisphere. (a) Find the total volume in terms of π. (b) Find the surface area of the curved surfaces (the silo stands on the ground), in terms of π. [5]



3.3 *(Original.)* A sphere fits exactly inside a closed cylinder (radius r). Show that the ratio of their volumes is $2:3$, and find the ratio of their surface areas. [4]



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3.2 (a) Cylinder $\pi(9)(7) = 63\pi$; hemisphere $\frac{2}{3}\pi(27) = 18\pi$; total 81π m^3. (b) Cylinder wall $2\pi(3)(7) = 42\pi$; hemisphere curved $2\pi(9) = 18\pi$; curved total 60π m^2 (the ground-covered base is excluded).

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3.3 Sphere $V = \frac{4}{3}\pi r^3$; cylinder $V = \pi r^2(2r) = 2\pi r^3$: ratio $\frac{4}{3} : 2 = 2 : 3$. ✓ Sphere $SA = 4\pi r^2$; cylinder total $= 2\pi r^2 + 2\pi r(2r) = 6\pi r^2$: ratio $4 : 6 = 2 : 3$.

Local review guidance: the right formula quoted before substitution; exact π forms; frustum parts label which cone each term belongs to. This is authored classroom guidance.

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