Exercise walk-through

3.2
Circle theorems with proof

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Angles, polygons and symmetry

- Angles at a point add up to $360°$; angles on a straight line add up to $180°$.
- **Vertically opposite** angles are equal.
- With parallel lines, **alternate angles** are equal, **corresponding angles** are equal, and co-interior angles add up to $180°$. Write these names, never "Z angles" or "F angles".
- The angles in a triangle add up to $180°$.

Pythagoras, trigonometry and 3D problems

- $c^2 = 9^2 + 11^2 - 2 \times 9 \times 11 \times \cos 72° = 202 - 61.2 = 140.8$, so $c = 11.9$ (3 s.f.).
- $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{121 + 140.8 - 81}{2 \times 11 \times 11.87} = 0.693$, so $A = 46.2°$.
- Check: $B = 180° - 72° - 46.2° = 61.8°$, and the smallest angle A is opposite the shortest side a.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \quad \cos\theta = \frac{\text{adj}}{\text{hyp}}, \quad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

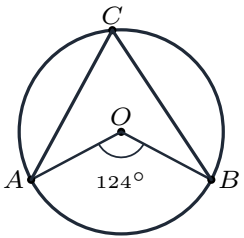
Transformations with matrices and vectors

- The first transformation goes on the right:
  $$\mathbf{RM} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.$$
- This is a reflection in the y-axis.
- Check with $(2, 1)$: the reflection gives $(1, 2)$, the rotation gives $(-2, 1)$, and a reflection in the y-axis also gives $(-2, 1)$.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

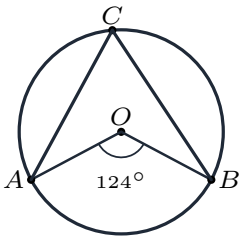
Exercise 2.1

2.1 O is the centre of the circle. Angle $AOB = 124°$. Find angle ACB. Give a reason. [2]



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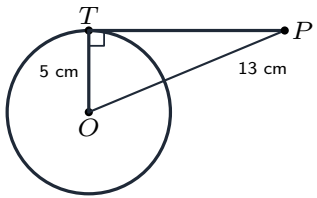


2.1

- angle $ACB = \mathbf{62}°$
- the angle at the centre is twice the angle at the circumference

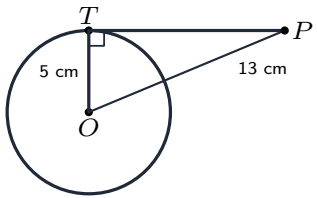
Exercise 2.2

2.2 TP is a tangent to the circle at T. O is the centre. $OT = 5$ cm and $OP = 13$ cm. Find the length of TP. [2]



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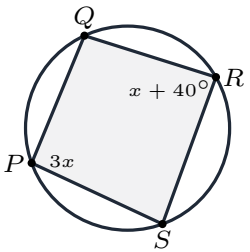


2.2

- angle $OTP = 90°$ (tangent and radius), so $TP^2 = 13^2 - 5^2$
- $TP = \sqrt{144} = \mathbf{12}$ cm

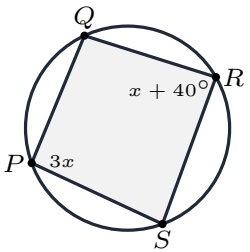
Exercise 2.3

2.3 $PQRS$ is a cyclic quadrilateral. Angle $P = 3x$ and angle $R = x + 40°$. Find x. [2]



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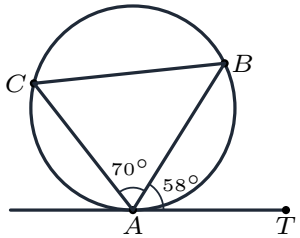


2.3

- $3x + x + 40 = 180$ (opposite angles of a cyclic quadrilateral)
- $x = \mathbf{35}°$

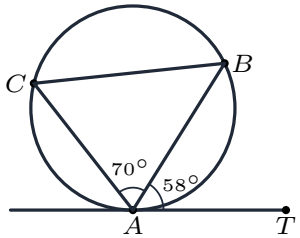
Exercise 3.1

3.1 A, B and C are points on a circle. The line TA is a tangent to the circle at A. Angle $TAB = 58°$ and angle $CAB = 70°$. Find angle ABC. Give reasons. [3]



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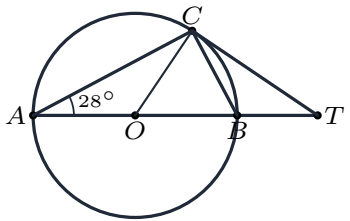
3.1

- angle $ACB = 58°$, alternate segment theorem
- angle $ABC = 180° - 58° - 70°$, angles in a triangle
- $= \mathbf{52°}$

Exercise 3.2 (a)

3.2 AB is a diameter of a circle with centre O. C is a point on the circle. The tangent at C meets the line AB, extended, at T. Angle $CAB = 28°$.

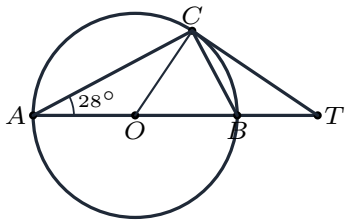
(a) Write down the size of angle ACB. Give a reason. [2]



Exercise 3.2 (a)

3.2 AB is a diameter of a circle with centre O. C is a point on the circle. The tangent at C meets the line AB, extended, at T. Angle $CAB = 28°$.

(a) Write down the size of angle ACB. Give a reason. [2]



(a) **90°**; the angle in a semicircle is 90°

Exercise 3.2 (b)

3.2 AB is a diameter of a circle with centre O. C is a point on the circle. The tangent at C meets the line AB, extended, at T. Angle $CAB = 28°$.

(b) Find angle ABC and angle OCB. Give reasons. [3]

Exercise 3.2 (b)

3.2 AB is a diameter of a circle with centre O. C is a point on the circle. The tangent at C meets the line AB, extended, at T. Angle $CAB = 28°$.

(b) Find angle ABC and angle OCB. Give reasons. [3]

(b) angle $ABC = 180° - 90° - 28° = \mathbf{62}°$, angles in a triangle; $OB = OC$ (radii), so triangle OBC is isosceles; angle $OCB = \mathbf{62}°$

Exercise 3.2 (c)

3.2 AB is a diameter of a circle with centre O. C is a point on the circle. The tangent at C meets the line AB, extended, at T. Angle $CAB = 28°$.

(c) Find angle CTB. Give reasons. [4]

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3.2 AB is a diameter of a circle with centre O. C is a point on the circle. The tangent at C meets the line AB, extended, at T. Angle $CAB = 28°$.

(c) Find angle CTB. Give reasons. [4]

(c) angle $OCT = 90°$, tangent and radius; angle $COB = 180° - 62° - 62° = 56°$; in triangle OCT: $180° - 90° - 56° = \mathbf{34}°$

Exercise 3.2 (d)

3.2 AB is a diameter of a circle with centre O. C is a point on the circle. The tangent at C meets the line AB, extended, at T. Angle $CAB = 28°$.

(d) Let angle $CAB = x$. Prove that angle $BCT = x$ for any position of C. Give a reason for each step. [5]

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(d) angle $ACB = 90°$, angle in a semicircle; angle $ABC = 90° - x$, angles in a triangle; angle $OCB = 90° - x$, triangle OBC is isosceles; angle $OCT = 90°$, tangent and radius; angle $BCT = 90° - (90° - x) = x$