Exercise walk-through

3.1 Angles, polygons, congruence and similarity

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Angles, polygons and symmetry

- Angles at a point add up to $360°$; angles on a straight line add up to $180°$.
- **Vertically opposite** angles are equal.
- With parallel lines, **alternate angles** are equal, **corresponding angles** are equal, and co-interior angles add up to $180°$. Write these names, never "Z angles" or "F angles".
- The angles in a triangle add up to $180°$.

Pythagoras, trigonometry and 3D problems

- $c^2 = 9^2 + 11^2 - 2 \times 9 \times 11 \times \cos 72° = 202 - 61.2 = 140.8$, so $c = 11.9$ (3 s.f.).
- $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{121 + 140.8 - 81}{2 \times 11 \times 11.87} = 0.693$, so $A = 46.2°$.
- Check: $B = 180° - 72° - 46.2° = 61.8°$, and the smallest angle A is opposite the shortest side a.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \quad \cos\theta = \frac{\text{adj}}{\text{hyp}}, \quad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

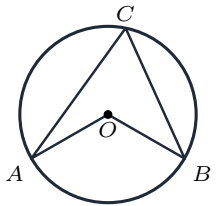
Transformations with matrices and vectors

- The first transformation goes on the right:
  $$\mathbf{RM} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.$$
- This is a reflection in the y-axis.
- Check with $(2, 1)$: the reflection gives $(1, 2)$, the rotation gives $(-2, 1)$, and a reflection in the y-axis also gives $(-2, 1)$.

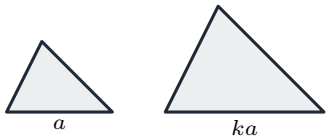
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

Exercises 2.1·2.2

2.1 A regular polygon has interior angle $150°$. Find n and the angle at the centre. [3]

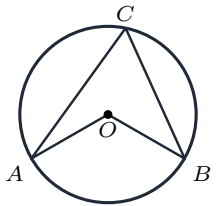


2.2 Two angles of a triangle are $47°$ and $71°$. Find the third and the exterior angle at that vertex. [2]



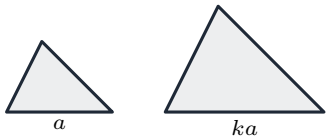
Exercises 2.1–2.2

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2.1 Exterior $= 30°$, so $n = 12$; centre angle $= 30°$.

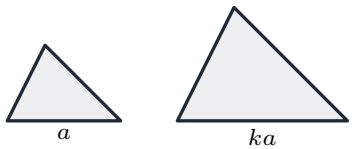
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2.2 Third angle $= 62°$; exterior $= 47 + 71 = 118°$.

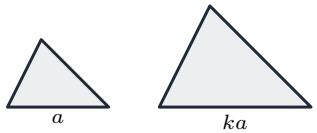
Setup for exercise 2.3

2.3 Write down the congruence condition for two triangles with two sides and the included angle equal. [1]

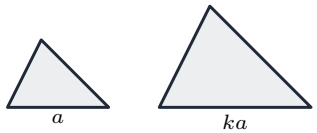


Exercise 2.3

2.4 Similar jugs have heights 8 cm and 12 cm. The smaller holds 144 cm^3. Find the larger's volume. [3]

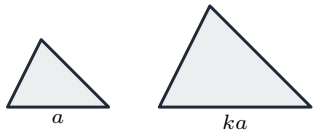


2.5 Similar photographs have areas 18 cm^2 and 50 cm^2. Find the ratio of their widths. [3]

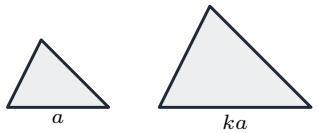


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2.3 SAS.

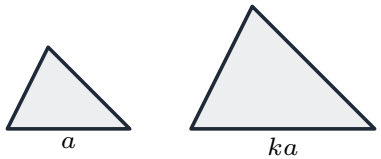
2.4 $k = \frac{12}{8} = \frac{3}{2}$; volumes scale by $\frac{27}{8}$: $144 \times \frac{27}{8} = 486$ cm^3.

2.5 $\sqrt{18} : \sqrt{50} = 3\sqrt{2} : 5\sqrt{2} = 3 : 5$.

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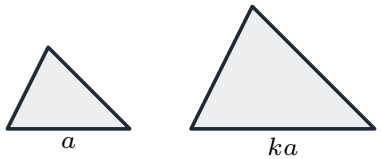
Exercise 3.1

3.1 *(Original, 2E SAS family.)* Triangles ABC and PQR have $AB = PQ = 7$ cm, $BC = QR = 9$ cm and $\angle ABC = \angle PQR = 54°$. (a) Prove the triangles are congruent. (b) Write down the length AC. [3]



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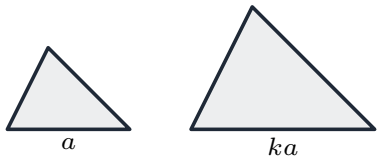
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3.1 (a) Two pairs of sides and the included angle are equal: SAS, so the triangles are congruent. (b) $AC = PR$ (corresponding sides) – but PR is given as 9 cm only if the correspondence maps $BC \to QR$; from SAS with $AB \leftrightarrow PQ$ and $BC \leftrightarrow QR$, the third sides correspond: $AC = PR$. With the data given, the cosine rule gives $AC^2 = 49 + 81 - 2(7)(9)\cos 54° = 130 - 126 \times 0.588 = 130 - 74.1 = 55.9$, so $AC = 7.5$ cm – and PR equals the same value by the same working.

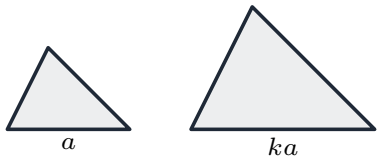
Exercise 3.2

3.2 *(Original, 2E similar-solids family.)* Two similar statues have heights in the ratio $3:5$; the smaller displaces $216\ \text{cm}^3$ of water. (a) Find the volume of the larger. (b) The larger's base area is $150\ \text{cm}^2$; find the smaller's. [4]



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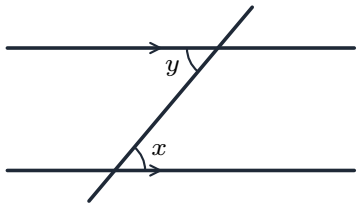
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3.2 (a) $k = \frac{5}{3}$; volume ratio $\frac{125}{27}$: $216 \times \frac{125}{27} = 1000\ \text{cm}^3$. (b) Area ratio $\frac{25}{9}$: smaller base $= 150 \times \frac{9}{25} = 54\ \text{cm}^2$.

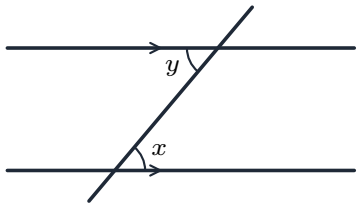
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3.3 Co-interior angles summing to 180° ($63° + 117°$) mean the two lines are parallel – the interior-angle property between parallels.

Local review guidance: the named fact before the number; exact values; congruence names the condition and the correspondence. This is authored classroom guidance.