Exercise walk-through

2.5
Equations, simultaneity and inequalities

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Straight-line graphs and coordinates

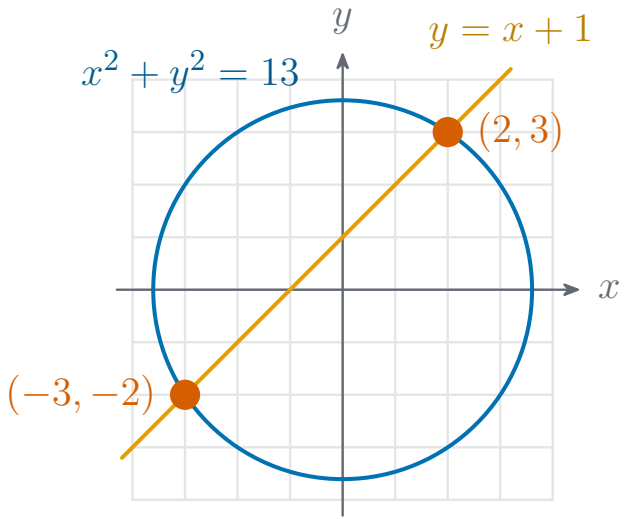
- Coordinates (x, y) can be in any of the four **quadrants** .
- The line $y = mx + c$ has gradient m and crosses the y-axis at $(0, c)$.
- (Extension) To find the equation of a line through two points, find the gradient first.
- Through $(1, 7)$ and $(2, 9)$: $m = \frac{9-7}{2-1} = 2$, then $7 = 2 \times 1 + c$, so $c = 5$ and $y = 2x + 5$.

Differentiation

- $\frac{dy}{dx} = 3x^2 - 12 = 0$, so $x^2 = 4$ and $x = \pm 2$.
- At $x = -2$: $y = -8 + 24 + 5 = 21$. The gradient goes from $+$ to $-$, so $(-2, 21)$ is a maximum.
- At $x = 2$: $y = 8 - 24 + 5 = -11$. The gradient goes from $-$ to $+$, so $(2, -11)$ is a minimum.
- At $x = 3$: $y = 27 - 36 + 5 = -4$ and the gradient is $3 \times 9 - 12 = 15$. The **tangent** is $y + 4 = 15(x - 3)$, so $y = 15x - 49$.

$$y = kx^n \implies \frac{dy}{dx} = knx^{n-1}$$

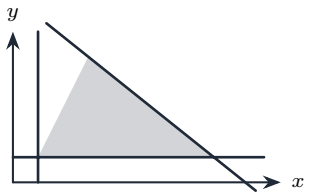
Figure: Equations and inequalities



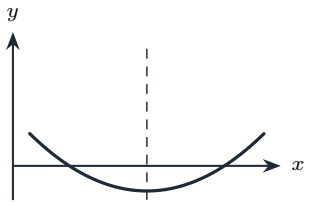
- $y = x + 1$ meets $x^2 + y^2 = 13$ twice
- Answers in pairs: $(2, 3)$ and $(-3, -2)$

Exercises 2.1·2.2

2.1 Solve $5x - 3 = 2x + 9$ [2]

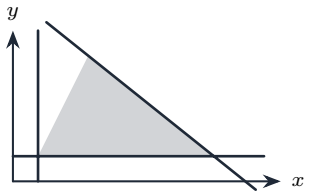


2.2 Solve $x^2 - 3x - 10 = 0$ [2]



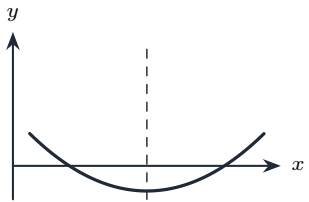
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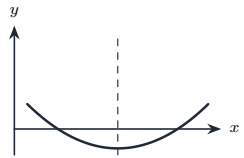
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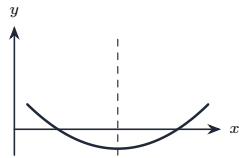
2.2 $(x - 5)(x + 2) = 0$: $x = 5$ or -2.

Exercise 2.3

2.3 Solve $x^2 + 6x + 2 = 0$, giving exact roots [3]



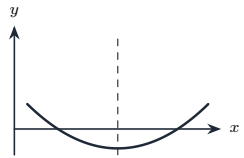
2.4 Solve the pair $y = 2x + 1$, $y = x^2 - 2$ [4]



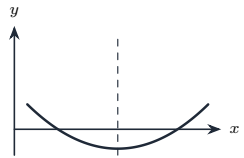
2.5 Solve $\frac{x}{3} + 2 > \frac{x}{2}$ [3]

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2.5 Solve $\frac{x}{3} + 2 > \frac{x}{2}$ [3]

2.3 $x = \frac{-6 \pm \sqrt{36 - 8}}{2} = \frac{-6 \pm \sqrt{28}}{2} = -3 \pm \sqrt{7}$.

2.4 $x^2 - 2 = 2x + 1$, so $x^2 - 2x - 3 = 0 = (x - 3)(x + 1)$: $x = 3$ or -1, giving $(3, 7)$ and

Full solution on next slide

Full solution 2.3

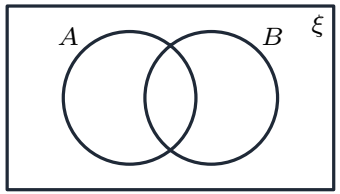
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2.5 Multiply by 6: $2x + 12 > 3x$, so $x < 12$.

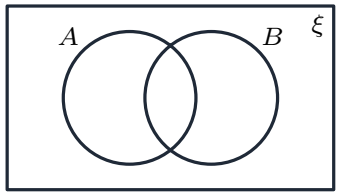
Exercise 3.1

3.1 *(Original, 1E intersection family.)* A line $y = 2x + 5$ meets the curve $y = x^2 + 1$ at two points. (a) Show the x-coordinates satisfy $x^2 - 2x - 4 = 0$. (b) Find both intersection points, exact where possible. [5]



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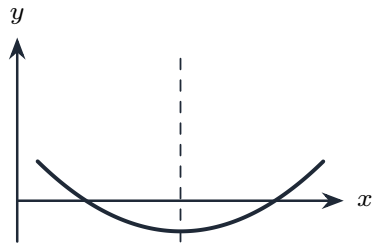


3.1 (a) $2x + 5 = x^2 + 1$ gives $x^2 - 2x - 4 = 0$. (b) $x = \dfrac{2 \pm \sqrt{4 + 16}}{2} = 1 \pm \sqrt{5}$; points $\left(1 + \sqrt{5},\, 7 + 2\sqrt{5}\right)$ and $\left(1 - \sqrt{5},\, 7 - 2\sqrt{5}\right)$.

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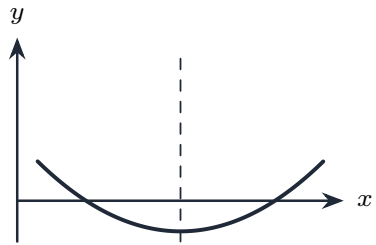
Exercise 3.2

3.2 *(Original.)* A rectangular field's length exceeds its width by 5 m; its area is 84 m^2. Form and solve a quadratic to find both dimensions. [4]



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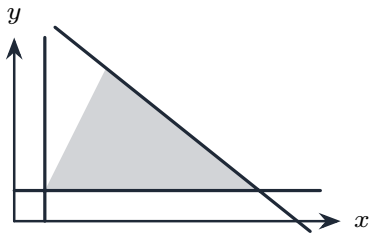
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3.2 Width w: $w(w+5)=84$, so $w^2+5w-84=0=(w+12)(w-7)$: width 7 m, length 12 m (reject -12).

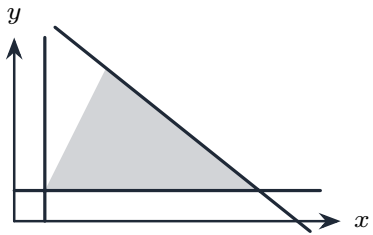
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3.3 *(Original, 2E inequality family.)* Solve and represent on a number line: $-7 \leq 3x+2 < 6$. [3]



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3.3 Subtract 2 throughout: $-9 \leq 3x < 4$; divide by 3: $-3 \leq x < \frac{4}{3}$; closed circle at -3, open at $\frac{4}{3}$.

Local review guidance: substitution shown before expansion; exact roots and both intersection points; context rejections justified. This is authored classroom guidance.