Exercise walk-through

2.3
Lines, curves and inequalities

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Straight-line graphs and coordinates

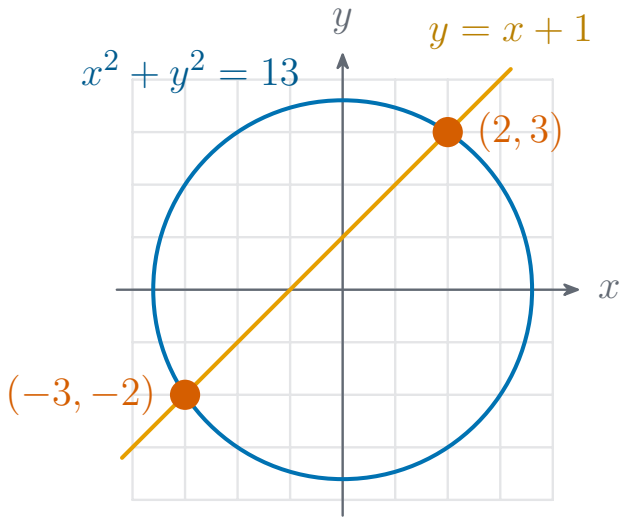
- Coordinates (x, y) can be in any of the four **quadrants** .
- The line $y = mx + c$ has gradient m and crosses the y-axis at $(0, c)$.
- (Extension) To find the equation of a line through two points, find the gradient first.
- Through $(1, 7)$ and $(2, 9)$: $m = \frac{9-7}{2-1} = 2$, then $7 = 2 \times 1 + c$, so $c = 5$ and $y = 2x + 5$.

Differentiation

- $\frac{dy}{dx} = 3x^2 - 12 = 0$, so $x^2 = 4$ and $x = \pm 2$.
- At $x = -2$: $y = -8 + 24 + 5 = 21$. The gradient goes from $+$ to $-$, so $(-2, 21)$ is a maximum.
- At $x = 2$: $y = 8 - 24 + 5 = -11$. The gradient goes from $-$ to $+$, so $(2, -11)$ is a minimum.
- At $x = 3$: $y = 27 - 36 + 5 = -4$ and the gradient is $3 \times 9 - 12 = 15$. The **tangent** is $y + 4 = 15(x - 3)$, so $y = 15x - 49$.

$$y = kx^n \implies \frac{dy}{dx} = knx^{n-1}$$

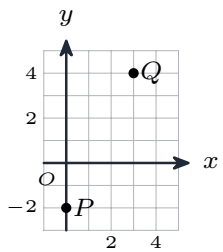
Figure: Equations and inequalities



- $y = x + 1$ meets $x^2 + y^2 = 13$ twice
- Answers in pairs: $(2, 3)$ and $(-3, -2)$

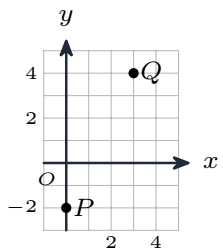
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2.1 Find the gradient of the line through P and Q, and its equation. [2]



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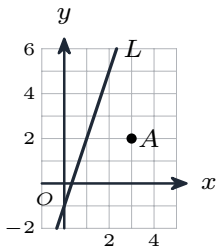


2.1

- gradient $= \frac{6}{3} = \mathbf{2}$
- $\mathbf{y = 2x - 2}$

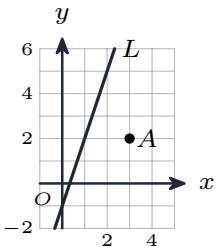
Exercise 2.2

2.2 The line L has equation $y = 3x - 1$. Write down the gradient of a line perpendicular to L. Find the equation of the line perpendicular to L through $A(3,\ 2)$. [2]



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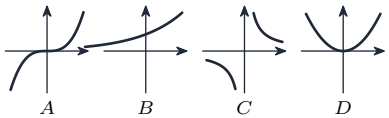


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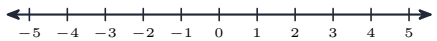
- $-\frac{1}{3}$
- $y - 2 = -\frac{1}{3}(x - 3)$, so $\mathbf{y = -\frac{1}{3}x + 3}$

Exercises 2.3·3.1

2.3 Match each sketch A to D with one of the equations $y = x^2$, $y = x^3$, $y = \dfrac{1}{x}$ and $y = 2^x$. [2]

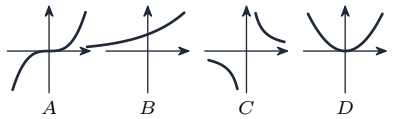


3.1 Solve $3(x - 2) < 2x + 5$. List the integers n with $-4 \leq 2n < 6$. [3]



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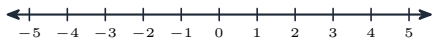
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2.3

- A: $y = x^3$; B: $y = 2^x$; C: $y = \dfrac{1}{x}$; D: $y = x^2$

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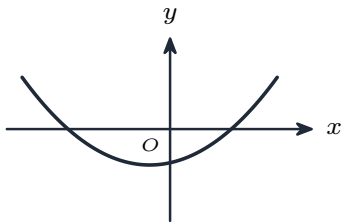
3.1

- $3x - 6 < 2x + 5$
- $\mathbf{x < 11}$
- $-2, -1, 0, 1, 2$

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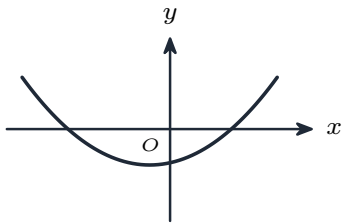
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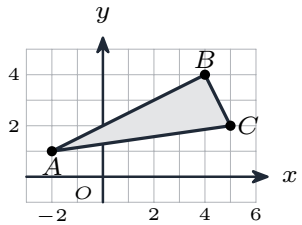
3.2

- $(x + 5)(x - 3)$
- roots -5 and 3
- $\mathbf{x \leq -5}$ **or** $\mathbf{x \geq 3}$

Exercise 3.3 (a)

3.3 A is $(-2,\ 1)$, B is $(4,\ 4)$ and C is $(5,\ 2)$.

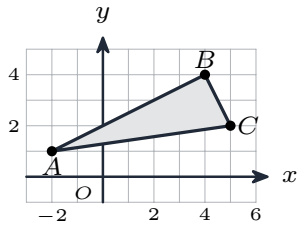
(a) Find the gradient of AB and the equation of the line AB. [3]



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(a) Find the gradient of AB and the equation of the line AB. [3]



(a) $\frac{4-1}{4+2} = \mathbf{\frac{1}{2}}$; $\mathbf{y = \frac{1}{2}x + 2}$

Exercise 3.3 (b)

3.3 A is $(-2,\ 1)$, B is $(4,\ 4)$ and C is $(5,\ 2)$.

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(b) gradient $BC = \frac{2-4}{5-4} = -2$; $\frac{1}{2} \times (-2) = -1$, so they are perpendicular

Exercise 3.3 (c)

3.3 A is $(-2,\ 1)$, B is $(4,\ 4)$ and C is $(5,\ 2)$.

(c) Find the lengths AB and BC as surds, and the area of triangle ABC. [4]

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(c) $AB = \sqrt{6^2 + 3^2} = \mathbf{3\sqrt{5}}$; $BC = \sqrt{1 + 4} = \mathbf{\sqrt{5}}$; the right angle is at B, so area $= \frac{1}{2} \times 3\sqrt{5} \times \sqrt{5} = \mathbf{7.5}$

Exercise 3.3 (d)

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(d) The triangle ABC, with its edges, is the region where three inequalities are true. Find the three inequalities. [5]

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(d) The triangle ABC, with its edges, is the region where three inequalities are true. Find the three inequalities. [5]

(d) $y \leq \frac{1}{2}x + 2$; line BC: $y = -2x + 12$, so $y \leq -2x + 12$; line AC: gradient $\frac{1}{7}$, $7y = x + 9$, so $7y \geq x + 9$