Exercise walk-through

2.2
Functions and proof

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Straight-line graphs and coordinates

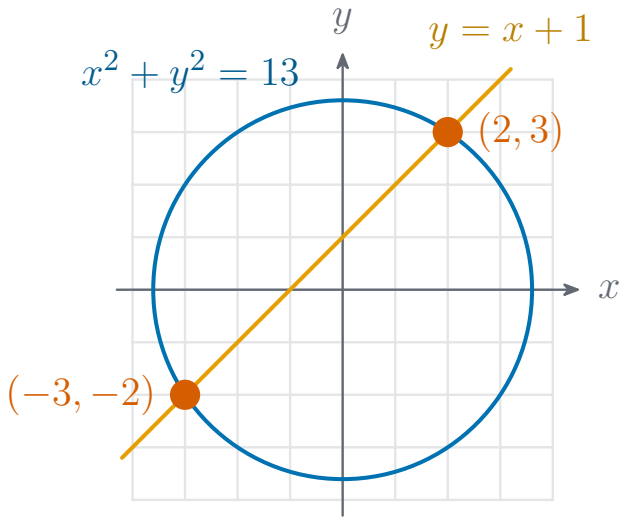
- Coordinates (x, y) can be in any of the four **quadrants** .
- The line $y = mx + c$ has gradient m and crosses the y-axis at $(0, c)$.
- (Extension) To find the equation of a line through two points, find the gradient first.
- Through $(1, 7)$ and $(2, 9)$: $m = \frac{9-7}{2-1} = 2$, then $7 = 2 \times 1 + c$, so $c = 5$ and $y = 2x + 5$.

Differentiation

- $\frac{dy}{dx} = 3x^2 - 12 = 0$, so $x^2 = 4$ and $x = \pm 2$.
- At $x = -2$: $y = -8 + 24 + 5 = 21$. The gradient goes from $+$ to $-$, so $(-2, 21)$ is a maximum.
- At $x = 2$: $y = 8 - 24 + 5 = -11$. The gradient goes from $-$ to $+$, so $(2, -11)$ is a minimum.
- At $x = 3$: $y = 27 - 36 + 5 = -4$ and the gradient is $3 \times 9 - 12 = 15$. The **tangent** is $y + 4 = 15(x - 3)$, so $y = 15x - 49$.

$$y = kx^n \implies \frac{dy}{dx} = knx^{n-1}$$

Figure: Equations and inequalities



- $y = x + 1$ meets $x^2 + y^2 = 13$ twice
- Answers in pairs: $(2, 3)$ and $(-3, -2)$

Exercises 2.1·2.2

2.1 $f(x) = 5 - 2x$. Find $f(-3)$. Solve $f(x) = -7$. [2]



2.2 $f(x) = \sqrt{x - 4}$. Write down the domain and the range of f. [2]



Exercises 2.1·2.2

2.1 $f(x) = 5 - 2x$. Find $f(-3)$. Solve $f(x) = -7$. [2]



2.1

- $f(-3) = \mathbf{11}$
- $5 - 2x = -7$, $x = \mathbf{6}$

2.2 $f(x) = \sqrt{x - 4}$. Write down the domain and the range of f. [2]



2.2

- domain: $x \geq 4$
- range: $f(x) \geq 0$

Exercises 2.3·3.1

2.3 Show that the statement on the card is false. [2]

$n^2 + n + 1$ is prime for every whole number n

3.1 Prove that the sum of any three consecutive integers is a multiple of 3. [3]

$7 + 8 + 9 = 24$ $22 + 23 + 24 = 69$

Exercises 2.3·3.1

2.3 Show that the statement on the card is false. [2]

$n^2 + n + 1$ is prime for every whole number n

2.3

- for example $n = 4$: $16 + 4 + 1 = 21$
- $21 = 3 \times 7$ is not prime, so the statement is false

3.1 Prove that the sum of any three consecutive integers is a multiple of 3. [3]

$7 + 8 + 9 = 24$ $22 + 23 + 24 = 69$

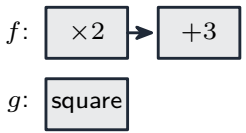
3.1

- $n + (n + 1) + (n + 2)$
- $= 3n + 3 = 3(n + 1)$
- $3(n + 1)$ has a factor of 3, so the sum is a multiple of 3

Exercise 3.2 (a)

3.2 $f(x) = 2x + 3$ and $g(x) = x^2$.

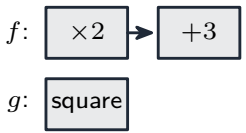
(a) Find $fg(4)$ and $gf(4)$. [2]



Exercise 3.2 (a)

3.2 $f(x) = 2x + 3$ and $g(x) = x^2$.

(a) Find $fg(4)$ and $gf(4)$. [2]



(a) $fg(4) = f(16) = \mathbf{35}$; $gf(4) = g(11) = \mathbf{121}$

Exercise 3.2 (b)

3.2 $f(x) = 2x + 3$ and $g(x) = x^2$.

(b) Find expressions for $fg(x)$ and $gf(x)$. [3]

Exercise 3.2 (b)

3.2 $f(x) = 2x + 3$ and $g(x) = x^2$.

(b) Find expressions for $fg(x)$ and $gf(x)$. [3]

(b) $fg(x) = \mathbf{2x^2 + 3}$; $gf(x) = (2x + 3)^2 = \mathbf{4x^2 + 12x + 9}$

Exercise 3.2 (c)

3.2 $f(x) = 2x + 3$ and $g(x) = x^2$.

(c) Solve $fg(x) = gf(x)$. Give your answers in the form $a \pm \sqrt{b}$. [4]

Exercise 3.2 (c)

3.2 $f(x) = 2x + 3$ and $g(x) = x^2$.

(c) Solve $fg(x) = gf(x)$. Give your answers in the form $a \pm \sqrt{b}$. [4]

(c) $2x^2 + 3 = 4x^2 + 12x + 9$; $x^2 + 6x + 3 = 0$; $(x + 3)^2 = 6$; $x = \mathbf{-3 \pm \sqrt{6}}$

Exercise 3.2 (d)

3.2 $f(x) = 2x + 3$ and $g(x) = x^2$.

(d) Find $f^{-1}(x)$. Prove that $f(n+1)^2 - f(n)^2$ – that is, $(2n+5)^2 - (2n+3)^2$ – is a multiple of 8 for every integer n. [5]

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(d) $x = 2y + 3$, $f^{-1}(x) = \dfrac{x-3}{2}$; $(4n^2 + 20n + 25) - (4n^2 + 12n + 9) = 8n + 16 = 8(n+2)$, which is a multiple of 8