Exercise walk-through

2.1 Expanding, factorising and algebraic fractions

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Straight-line graphs and coordinates

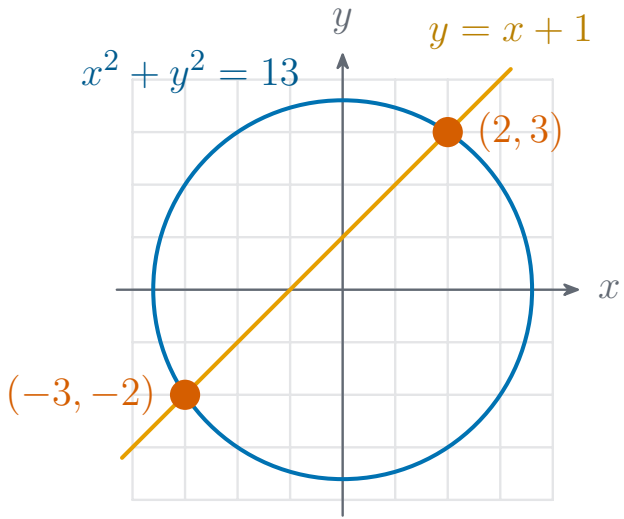
- Coordinates (x, y) can be in any of the four **quadrants** .
- The line $y = mx + c$ has gradient m and crosses the y-axis at $(0, c)$.
- (Extension) To find the equation of a line through two points, find the gradient first.
- Through $(1, 7)$ and $(2, 9)$: $m = \frac{9-7}{2-1} = 2$, then $7 = 2 \times 1 + c$, so $c = 5$ and $y = 2x + 5$.

Differentiation

- $\frac{dy}{dx} = 3x^2 - 12 = 0$, so $x^2 = 4$ and $x = \pm 2$.
- At $x = -2$: $y = -8 + 24 + 5 = 21$. The gradient goes from $+$ to $-$, so $(-2, 21)$ is a maximum.
- At $x = 2$: $y = 8 - 24 + 5 = -11$. The gradient goes from $-$ to $+$, so $(2, -11)$ is a minimum.
- At $x = 3$: $y = 27 - 36 + 5 = -4$ and the gradient is $3 \times 9 - 12 = 15$. The **tangent** is $y + 4 = 15(x - 3)$, so $y = 15x - 49$.

$$y = kx^n \implies \frac{dy}{dx} = knx^{n-1}$$

Figure: Equations and inequalities



- $y = x + 1$ meets $x^2 + y^2 = 13$ twice
- Answers in pairs: $(2, 3)$ and $(-3, -2)$

Exercises 2.1·2.2

2.1 Expand and simplify the two expressions. [2]

$(x+4)(x-6)$ $(2x-3)^2$

2.2 Factorise fully the two expressions. [2]

x^2-49 $3x^2-12x$

Exercises 2.1·2.2

2.1 Expand and simplify the two expressions. [2]

$(x+4)(x-6)$ $(2x-3)^2$

2.1

- $\mathbf{x^2 - 2x - 24}$
- $\mathbf{4x^2 - 12x + 9}$

2.2 Factorise fully the two expressions. [2]

$x^2 - 49$ $3x^2 - 12x$

2.2

- $\mathbf{(x+7)(x-7)}$
- $\mathbf{3x(x-4)}$

Exercises 2.3·3.1

2.3 Factorise $3x^2 - 10x - 8$. [2]

$3x^2 - 10x - 8$

3.1 Simplify fully the algebraic fraction. [3]

$$\frac{x^2 - 9}{x^2 + x - 6}$$

Exercises 2.3·3.1

2.3 Factorise $3x^2 - 10x - 8$. [2]

$\boxed{3x^2 - 10x - 8}$

2.3

- $3x^2 - 12x + 2x - 8$
- $(\mathbf{3x + 2})(\mathbf{x - 4})$

3.1 Simplify fully the algebraic fraction. [3]

$\boxed{\dfrac{x^2 - 9}{x^2 + x - 6}}$

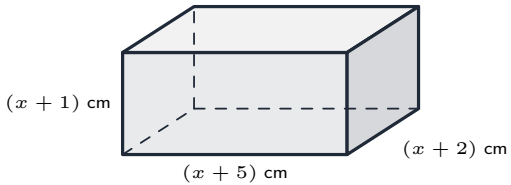
3.1

- top: $(x + 3)(x - 3)$
- bottom: $(x + 3)(x - 2)$
- $\dfrac{\mathbf{x - 3}}{\mathbf{x - 2}}$

Exercise 3.2 (a)

3.2 Box A is a cuboid with edges $(x+1)$ cm, $(x+2)$ cm and $(x+5)$ cm. Lengths are in cm.

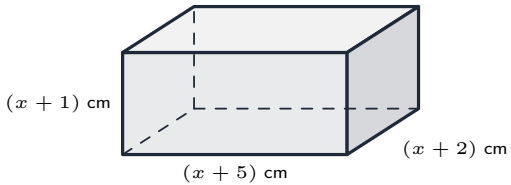
(a) Show that the area of the base is $x^2 + 3x + 2$ cm^2. [2]



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(a) $(x+1)(x+2) = x^2+2x+x+2 = x^2+3x+2$

Exercise 3.2 (b)

3.2 Box A is a cuboid with edges $(x+1)$ cm, $(x+2)$ cm and $(x+5)$ cm. Lengths are in cm.

(b) Show that the volume of box A is $x^3 + 8x^2 + 17x + 10$ cm^3. [3]

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3.2 Box A is a cuboid with edges $(x+1)$ cm, $(x+2)$ cm and $(x+5)$ cm. Lengths are in cm.

(b) Show that the volume of box A is $x^3+8x^2+17x+10$ cm^3. [3]

(b) $(x^2+3x+2)(x+5)$; $x^3+5x^2+3x^2+15x+2x+10$; $x^3+8x^2+17x+10$

Exercise 3.2 (c)

3.2 Box A is a cuboid with edges $(x+1)$ cm, $(x+2)$ cm and $(x+5)$ cm. Lengths are in cm.

(c) Find an expression for the total surface area of box A. Simplify your answer. [4]

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(c) faces $(x+1)(x+2)$, $(x+1)(x+5)$, $(x+2)(x+5)$; $x^2+3x+2+x^2+6x+5+x^2+7x+10 = 3x^2+16x+17$; total $\mathbf{6x^2+32x+34}$ cm^2

Exercise 3.2 (d)

3.2 Box A is a cuboid with edges $(x+1)$ cm, $(x+2)$ cm and $(x+5)$ cm. Lengths are in cm.

(d) Box B has every edge 1 cm shorter than box A. Show that box A has $3x^2+13x+10$ cm^3 more volume than box B. The difference is 48 cm^3. Find x. [5]

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(d) box B: $x(x+1)(x+4) = x^3+5x^2+4x$; difference $3x^2+13x+10$; $3x^2+13x-38=0$, $(x-2)(3x+19)=0$; $x = \mathbf{2}$