Course notes

OxfordAQA · International GCSE · Mathematics

9260: course teaching notes

Version: Version 5.1; teaching 2016, assessment 2018 onwards

These are original teaching notes, and each lesson is labelled with its tier.

3.1 · Exact arithmetic and estimation · Core

Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor** 质因数: A prime number that divides the integer exactly.

The method

- Prime factors reveal shared structure.
- Use the smallest common prime powers for the HCF and the largest for the LCM.
- Estimate before calculating; use brackets to preserve the order of operations.

A worked case

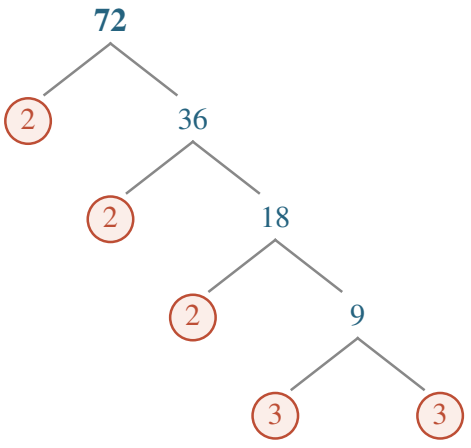
$$72 = 2^3 \times 3^2, \quad 90 = 2 \times 3^2 \times 5, \quad \text{HCF} = 18$$

■ Worked example

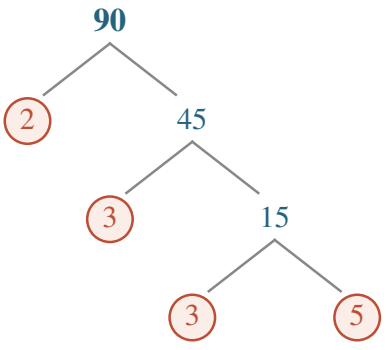
72=2^3×3^2 and 90=2×3^2×5. Their HCF is 2×9=18. Make 18 bags with 4 pencils and 5 pens each. Their LCM is 2^3×3^2×5=360.

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Factor trees: HCF(72, 90) = $2 \times 3^2 = 18$



$72 = 2^3 \times 3^2$



$90 = 2 \times 3^2 \times 5$

Where it goes wrong

- The HCF divides both numbers; the LCM is a multiple of both.
- They answer different questions.
- A decimal estimate is not an exact fraction.

■ Watch out

Not true: The HCF of two positive integers is larger than either integer. The HCF divides both numbers, so it can never be larger than the smaller one: the HCF of 72 and 90 is 18.

Applying it

- For a non-calculator paper, keep fractions exact and show cancellation.
- For a calculator paper, enter the full expression and compare with your estimate.

In your course

- OxfordAQA International GCSE Mathematics (9260), Core tier, specification section 3.1.

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■ **Key idea**

- **Prime factor**: A prime number that divides the integer exactly.
- Prime factors reveal shared structure.
- Watch for: the HCF divides both numbers; the LCM is a multiple of both.

3.1 · Exact arithmetic and estimation · Extension

Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor** 质因数: A prime number that divides the integer exactly.

The method

- Prime factors reveal shared structure.
- Use the smallest common prime powers for the HCF and the largest for the LCM.
- Estimate before calculating; use brackets to preserve the order of operations.

A worked case

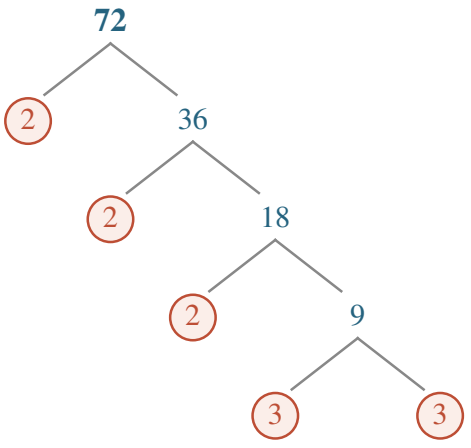
$$a = \prod p_i^{\alpha_i}, \quad b = \prod p_i^{\beta_i}, \quad \mathrm{HCF}(a,b) = \prod p_i^{\min(\alpha_i,\beta_i)}$$

■ **Worked example**

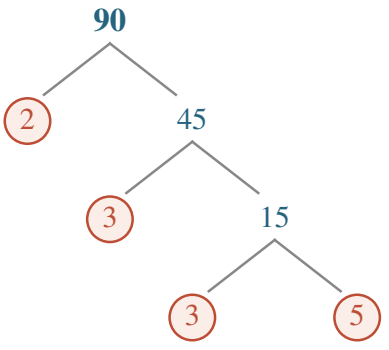
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> **Not true:** The HCF of two positive integers is larger than either integer. The HCF divides both numbers, so it can never be larger than the smaller one: the HCF of 72 and 90 is 18.

Applying it

- For a non-calculator paper, keep fractions exact and show cancellation.
- For a calculator paper, enter the full expression and compare with your estimate.

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■ **Key idea**

- **Prime factor**: A prime number that divides the integer exactly.
- Prime factors reveal shared structure.
- Watch for: the HCF divides both numbers; the LCM is a multiple of both.

3.1 · Integer indices and standard form · Core

How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
- This lesson studies **index** 指数: The power to which a base is raised.

The method

- Use integer powers, square and cube roots and standard form with $1 \leq a < 10$.
- In multiplying powers with the same base, add indices; in division, subtract them.

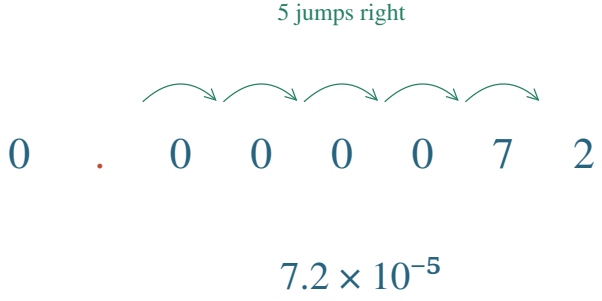
A worked case

$$a^m a^n = a^{m+n}$$

■ **Worked example**

0.000072=7.2×10^(-5). Also $2^3 \times 2^4 = 2^7 = 128$. The square root of 81 is 9. Check a standard-form answer by writing it out as a decimal.

$0.000072 = 7.2 \times 10^{-5}$: the point moves 5 places



Where it goes wrong

- Index laws do not turn a sum into a single power: 2^3+2^4=24, not 2^7.
- Do not round a surd when an exact answer is requested.

> ■ **Watch out**
>
> **Not true:** For every positive a, a^2+a^3 equals a^5. Index laws only add powers when the terms are multiplied: $2^2+2^3=12$, but $2^5=32$.

Applying it

- This Foundation/Core lesson excludes fractional powers and surd rationalisation.
- Estimate a result before using a calculator and retain the required precision.

In your course

- OxfordAQA International GCSE Mathematics (9260), Core tier, specification section 3.1.

■ **Key idea**

- **Index**: The power to which a base is raised.
- Use integer powers, square and cube roots and standard form with 1≤a<10.
- Watch for: index laws do not turn a sum into a single power: 2^3+2^4=24, not 2^7.

3.1 · Indices, surds and standard form · Extension

How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
- This lesson studies **index** 指数: The power to which a base is raised.

The method

- For the same positive base, multiplication adds indices and division subtracts them.
- A negative index means reciprocal; a fractional index represents a root.
- Standard form has 1≤a<10.

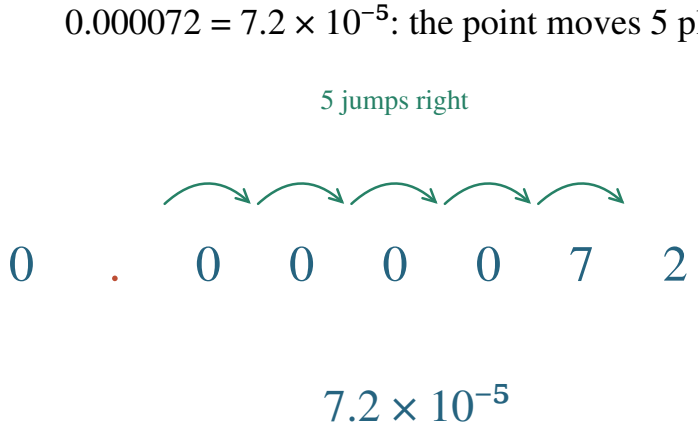
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A worked case

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = \left(\sqrt[n]{a}\right)^m$$

■ Worked example

0.000072=7.2×10^(-5). Also 16^(3/4)=(16^(1/4))^3=2^3=8. Simplify √72=6√2, then rationalise 1/√2=√2/2.



Where it goes wrong

- Index laws do not turn a sum into a single power: 2^3+2^4=24, not 2^7.
- Do not round a surd when an exact answer is requested.

■ Watch out

Not true: For every positive a, a^2+a^3 equals a^5. Index laws only add powers when the terms are multiplied: $2^2+2^3=12$, but $2^5=32$.

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Applying it

- Check powers of ten against the original quantity.
- Use surds for exact geometry, and round only the final length when the question asks for a decimal.

In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Index**: The power to which a base is raised.
- For the same positive base, multiplication adds indices and division subtracts them.
- Watch for: index laws do not turn a sum into a single power: 2^3+2^4=24, not 2^7.

3.1 · Percentages, ratio and proportional reasoning · Core

Can we recover the original price?

- A coat is reduced by 20% to ¥240. The discount applies to the original price, not to the sale price.
- This lesson studies **multiplier** 乘数: A factor that performs a percentage change in one multiplication.

The method

- A p% increase has multiplier 1+p/100; a decrease has multiplier 1-p/100.
- Reverse a percentage by dividing by the multiplier.
- In a ratio, first find the total number of parts.

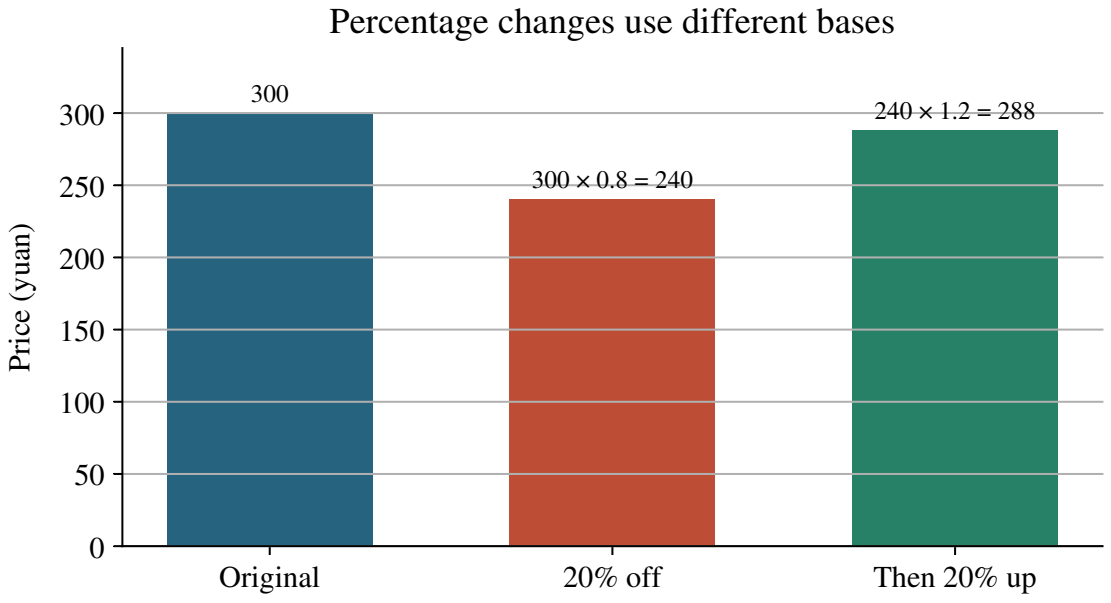
A worked case

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100}\right)$$

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■ Worked example

Let the original price be P. The model is sale price=0.8P. Hence P=240/0.8=300. A later 20% increase gives 240×1.2=288, so the two changes do not cancel.



Where it goes wrong

- A percentage uses a stated base.
- Subtracting the percentages loses that base.
- For compound change, multiply the multipliers; do not add the percentages.

■ Watch out

Not true: A 20% decrease followed by a 20% increase restores the starting price. The 20% increase is taken of a smaller amount: 100×0.8×1.2=96, not 100.

Applying it

- Use percentage multipliers and divide a total into ratio parts.
- This Foundation/Core lesson uses linear proportional contexts, not the advanced regression methods.

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In your course

- OxfordAQA International GCSE Mathematics (9260), Core tier, specification section 3.1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Multiplier**: A factor that performs a percentage change in one multiplication.
- A p% increase has multiplier 1+p/100; a decrease has multiplier 1-p/100.
- Watch for: A percentage uses a stated base.

3.1 · Percentages, ratio and proportional reasoning · Extension

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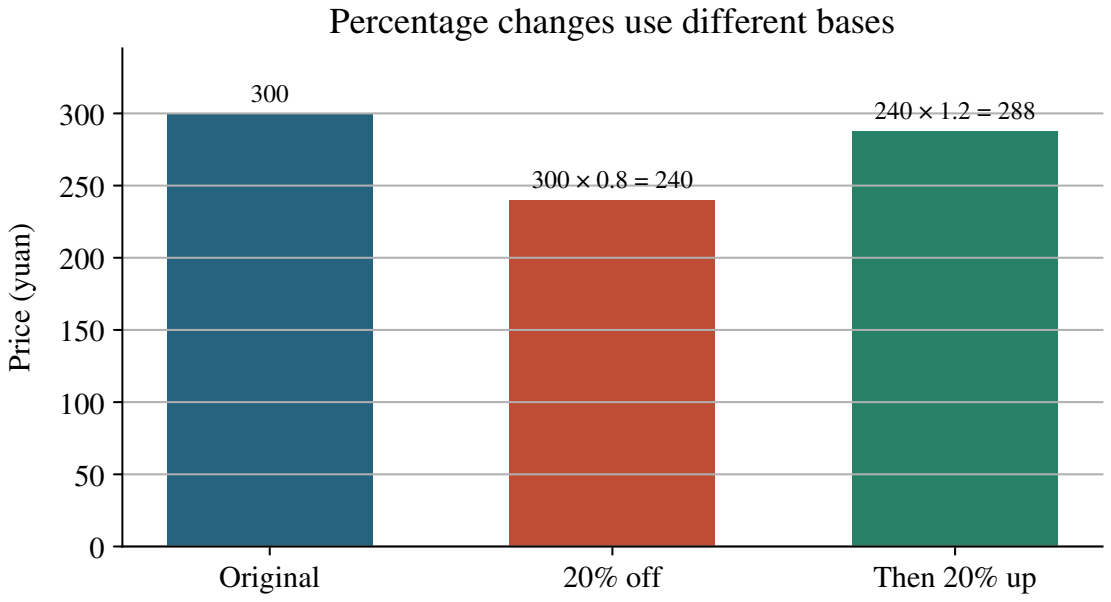
- A p% increase has multiplier 1+p/100; a decrease has multiplier 1-p/100.
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A worked case

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Let the original price be P. The model is sale price=0.8P. Hence P=240/0.8=300. A later 20% increase gives 240×1.2=288, so the two changes do not cancel.



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> **Not true:** A 20% decrease followed by a 20% increase restores the starting price. The 20% increase is taken of a smaller amount: $100\times0.8\times1.2=96$, not 100.

Applying it

- For direct proportion use y=kx; for inverse proportion use y=k/x.
- Calculate k from a known pair before using a new value.
- State what you held constant.

In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Multiplier**: A factor that performs a percentage change in one multiplication.
- A p% increase has multiplier 1+p/100; a decrease has multiplier 1-p/100.
- Watch for: A percentage uses a stated base.

3.1 · Accuracy, bounds and compound measures · Extension

Is the printed measurement exact?

- A rectangular panel is labelled 8.0 cm by 5.0 cm, each to the nearest 0.1 cm. Its true area is not fixed at 40 cm^2.
- This lesson studies **lower bound** 下界: The smallest possible value consistent with a stated rounding rule.

The method

- A value rounded to the nearest unit u lies from stated value-u/2 up to, but usually not including, stated value+u/2.
- For positive quantities, combine extremes according to the operation.

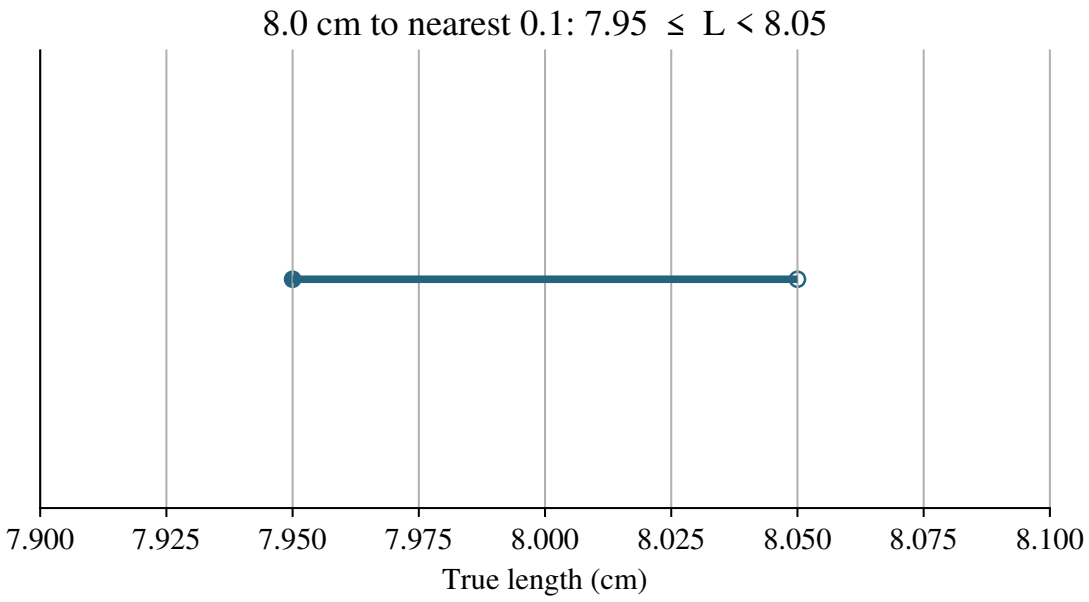
A worked case

$$A = LW, \qquad v = \frac{d}{t}$$

■ **Worked example**

The lengths satisfy 7.95≤L<8.05 and 4.95≤W<5.05. Since A=LW, 39.3525≤A<40.6525. For speed d/t, the largest speed uses the largest distance and smallest positive time.

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Where it goes wrong

- An upper bound is not automatically achieved.
- Dividing upper distance by upper time does not give the largest speed.
- Keep enough digits in intermediate calculations.

■ Watch out

Not true: The greatest value of positive d/t uses the greatest d and greatest t. To make d/t as large as possible, use the greatest d and the least t; a larger t makes the quotient smaller.

Applying it

- Distinguish measurement uncertainty from arithmetic rounding.
- A sensible reported precision cannot be finer than the measurements justify.

In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.1.
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■ **Key idea**

- **Lower bound**: The smallest possible value consistent with a stated rounding rule.
- A value rounded to the nearest unit u lies from stated value-u/2 up to, but usually not including, stated value+u/2.
- Watch for: an upper bound is not automatically achieved.

3.2 · Equations, identities and rearrangement · Core

When do two plans cost the same?

- Two mobile plans cost 20+3x and 44+x yuan for x GB. When do they cost the same?
- This lesson studies **identity** 恒等式: An equality that holds for every allowed value of its variable.

The method

- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs.
- Preserve equality by applying the same operation to both sides.
- State restrictions before dividing by a variable.

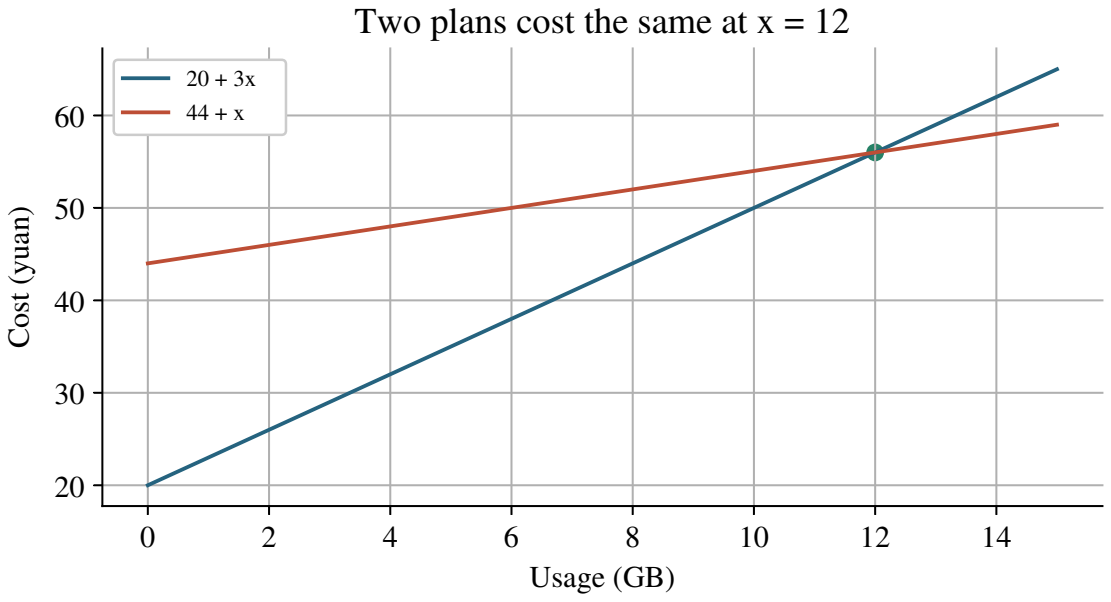
A worked case

$$C_1 = 20 + 3x, \qquad C_2 = 44 + x, \qquad C_1 = C_2$$

■ **Worked example**

20+3x=44+x gives 2x=24 and x=12. Both plans then cost 56. In A=πr^2, divide by π and take the positive square root to obtain r=$\sqrt{}$(A/π), because r is a length.

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Where it goes wrong

- Cancelling a term is not the same as cancelling a factor.
- In $(x^2+2x)/x$, factor the numerator and retain $x \neq 0$.
- Check a rearrangement by substitution.

> ■ **Watch out**
>
> **Not true:** Cancelling x from (x+3)/x leaves 3 for every nonzero x. x is a term of the numerator, not a factor, so it cannot be cancelled: at x=1, (1+3)/1=4, not 3.

Applying it

- Define the unknown and set up a linear equation.
- Check the answer by substitution.
- Restrict this Foundation/Core lesson to simple expressions and equations.

In your course

- OxfordAQA International GCSE Mathematics (9260), Core tier, specification section 3.2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ **Key idea**

- **Identity**: An equality that holds for every allowed value of its variable.
- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs.
- Watch for: cancelling a term is not the same as cancelling a factor.

3.2 · Equations, identities and rearrangement · Extension

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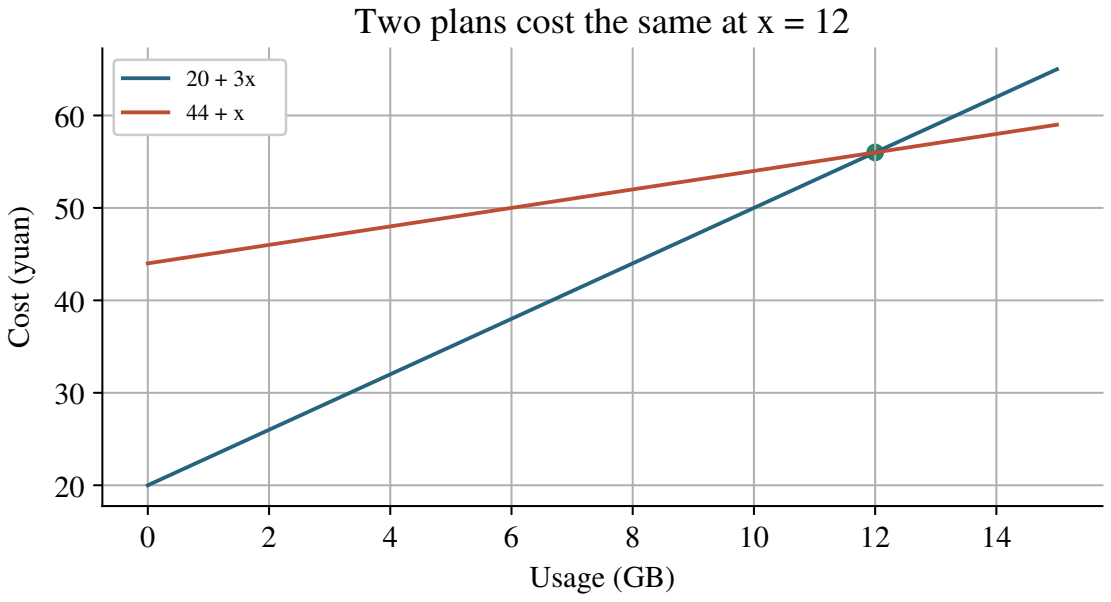
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> **Not true:** Cancelling x from (x+3)/x leaves 3 for every nonzero x. x is a term of the numerator, not a factor, so it cannot be cancelled: at x=1, (1+3)/1=4, not 3.

Applying it

- Set up the equation from units and the meaning of the unknown.
- A negative or fractional solution may be algebraically correct but impossible for a count.

In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Identity**: An equality that holds for every allowed value of its variable.
- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs.
- Watch for: cancelling a term is not the same as cancelling a factor.

3.2 · Factorising and solving simple quadratics · Core

Which widths make enough space?

- A rectangular enclosure has area x(10-x). What widths give at least 21 square metres?
- This lesson studies **discriminant** 判别式: The quantity b^2-4ac that determines the real roots of ax^2+bx+c=0.

The method

- Expand brackets and factorise simple quadratics.
- Solve by setting each factor equal to zero, and use a graph to interpret the roots.

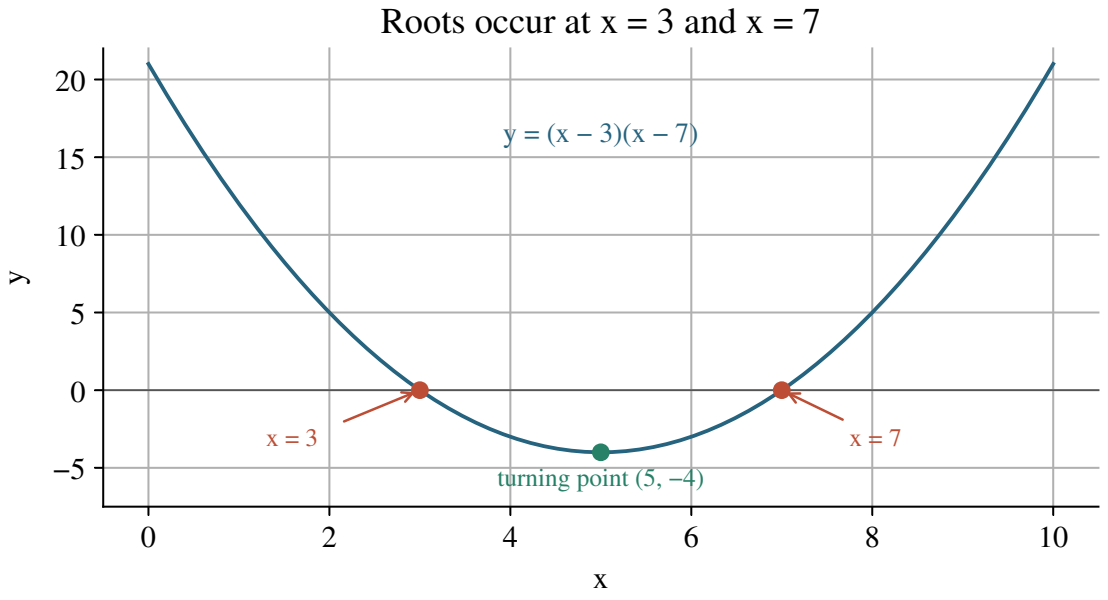
A worked case

$$(x-3)(x-7)=0$$

■ **Worked example**

x^2-10x+21=(x-3)(x-7). Hence the equation x^2-10x+21=0 has roots 3 and 7. Check each root by substitution and mark both intercepts on the graph.

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Where it goes wrong

- Multiplying an inequality by a negative number reverses its direction.
- A sketch must show which side of each root satisfies the inequality.
- Geometry may restrict x further.

> ■ **Watch out**
>
> **Not true:** A positive discriminant means that a quadratic has no real roots. A positive discriminant means two distinct real roots; a negative discriminant means no real roots.

Applying it

- This Foundation/Core lesson uses factorisation and graphical roots; the discriminant, quadratic formula and quadratic inequalities are reserved for the advanced tier.

In your course

- OxfordAQA International GCSE Mathematics (9260), Core tier, specification section 3.2.

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- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Discriminant**: The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$.
- Expand brackets and factorise simple quadratics.
- Watch for: multiplying an inequality by a negative number reverses its direction.

3.2 · Quadratics and inequalities · Extension

Which widths make enough space?

- A rectangular enclosure has area x(10-x). What widths give at least 21 square metres?
- This lesson studies **discriminant** 判别式: The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$.

The method

- Factor where possible; otherwise complete the square or use the quadratic formula.
- A quadratic inequality needs the sign on intervals, not only the roots.
- The discriminant identifies repeated or missing real roots.

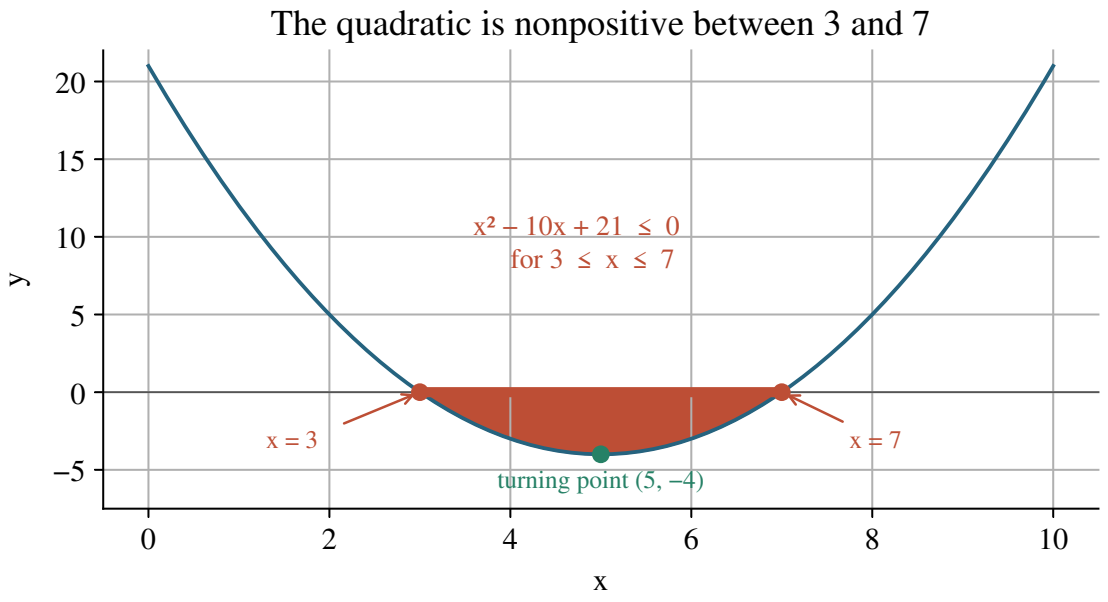
A worked case

$$ax^2 + bx + c = 0, \qquad \Delta = b^2 - 4ac$$

Worked example

The condition is $x(10-x)\geq 21$, so $x^2-10x+21\leq 0$. Factor $(x-3)(x-7)\leq 0$. The upward parabola is nonpositive between its roots, giving $3\leq x\leq 7$. The maximum area is 25 at $x=5$.

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Where it goes wrong

- Multiplying an inequality by a negative number reverses its direction.
- A sketch must show which side of each root satisfies the inequality.
- Geometry may restrict x further.

> **Watch out**
>
> **Not true:** A positive discriminant means that a quadratic has no real roots. A positive discriminant means two distinct real roots; a negative discriminant means no real roots.

Applying it

- Use the vertex to interpret an optimum.
- Check an endpoint and a point between the roots; the algebra and graph should tell the same story.

In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.2.

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- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Discriminant**: The quantity b^2-4ac that determines the real roots of ax^2+bx+c=0.
- Factor where possible; otherwise complete the square or use the quadratic formula.
- Watch for: multiplying an inequality by a negative number reverses its direction.

3.2 · Straight lines and gradients · Core

How does a path's slope become an equation?

- A path rises 6 metres over a horizontal distance of 3 metres. Its gradient connects a diagram to an equation.
- This lesson studies **gradient** 斜率: The change in y divided by the corresponding change in x.

The method

- Gradient is change in y divided by change in x.
- A straight line has y=mx+c, where c is its y-intercept.
- Parallel lines have equal gradients.

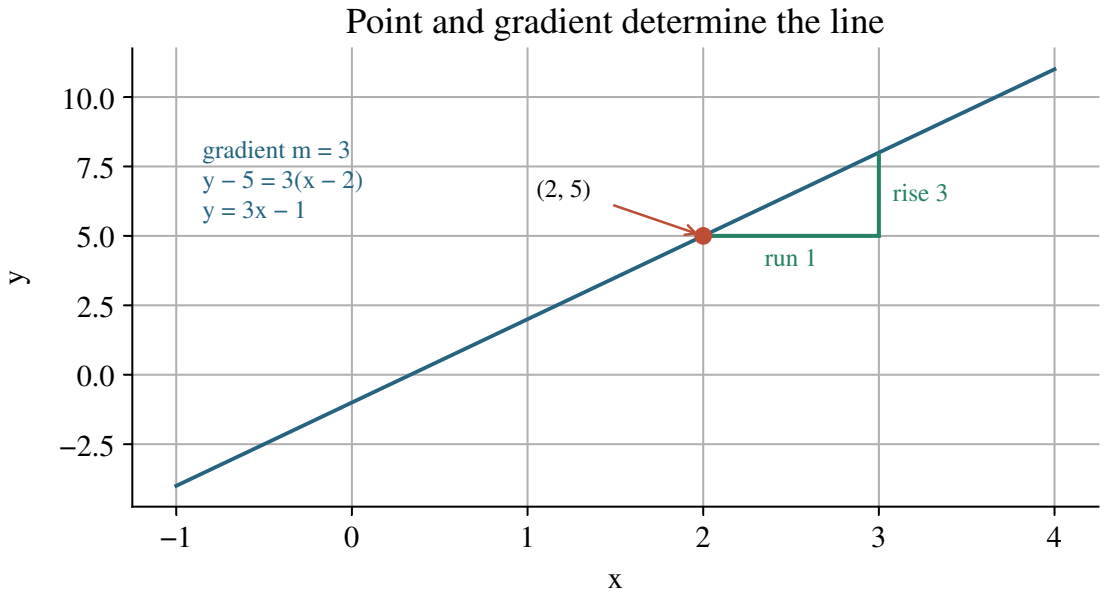
A worked case

$$y = mx + c$$

■ Worked example

Through (2,5) with gradient 3, substitute to get 5=3×2+c, so c=-1 and y=3x-1. Points (1,2) and (4,8) give gradient (8-2)/(4-1)=2.

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Where it goes wrong

- A vertical line has no finite gradient; do not force it into y=mx+c.
- Read the signs of a circle's centre carefully.
- The radius to a tangent is perpendicular to the tangent.

■ Watch out

Not true: Perpendicular nonvertical lines have equal gradients. Perpendicular gradients multiply to -1: a gradient of 2 needs $-1/2$. Parallel lines have equal gradients.

Applying it

- Plot a straight line using two checked points and label its intercept.
- Perpendicular-gradient formulae and circle equations are not part of this Foundation/Core lesson.

In your course

- OxfordAQA International GCSE Mathematics (9260), Core tier, specification section 3.2.

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- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Gradient**: The change in y divided by the corresponding change in x.
- Gradient is change in y divided by change in x.
- Watch for: A vertical line has no finite gradient; do not force it into y=mx+c.

3.2 · Coordinate geometry and tangents · Extension

How does a path's slope become an equation?

- A path rises 6 metres over a horizontal distance of 3 metres. Its gradient connects a diagram to an equation.
- This lesson studies **gradient** 斜率: The change in y divided by the corresponding change in x.

The method

- A line through (x_1,y_1) with gradient m has $y-y_1=m(x-x_1)$.
- Parallel lines have equal gradients.
- Finite perpendicular gradients multiply to -1.
- A circle has $(x-a)^2+(y-b)^2=r^2$.

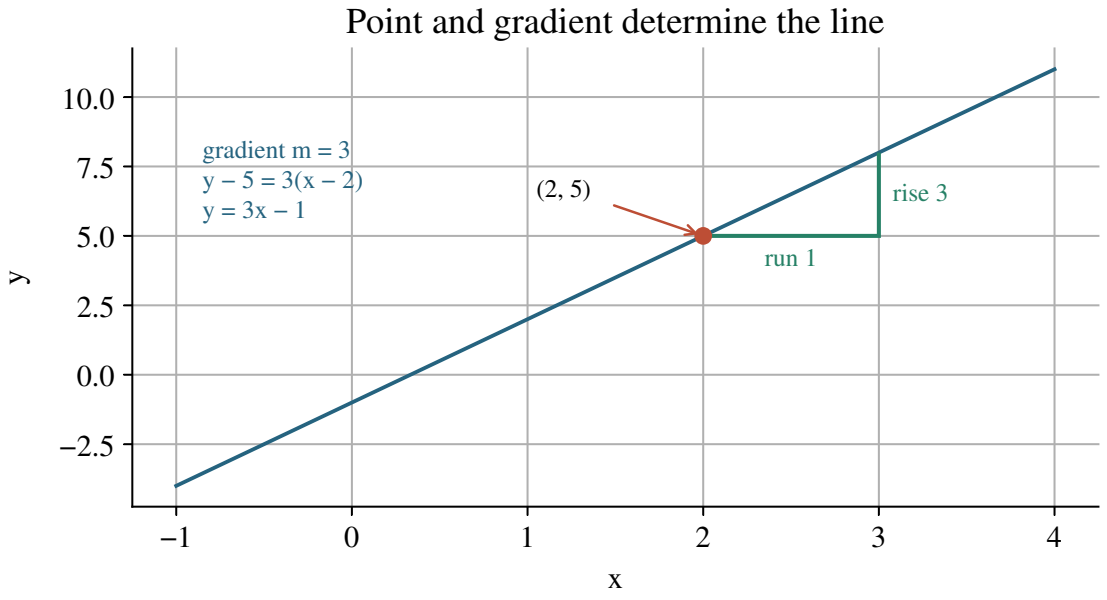
A worked case

$$y - y_1 = m(x - x_1), \qquad (x - a)^2 + (y - b)^2 = r^2$$

■ Worked example

Through (2,5) with gradient 3, y-5=3(x-2), so y=3x-1. A perpendicular through the same point has y-5=-(x-2)/3. The circle $(x-2)^2+(y+1)^2=25$ has centre (2,-1) and radius 5.

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Where it goes wrong

- A vertical line has no finite gradient; do not force it into y=mx+c.
- Read the signs of a circle's centre carefully.
- The radius to a tangent is perpendicular to the tangent.

■ Watch out

Not true: Perpendicular nonvertical lines have equal gradients. Perpendicular gradients multiply to -1: a gradient of 2 needs $-1/2$. Parallel lines have equal gradients.

Applying it

- Before solving a line-circle intersection, predict whether there are zero, one or two intersections.
- Substitution produces a quadratic whose discriminant checks the prediction.

In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.2.

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- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ **Key idea**

- **Gradient**: The change in y divided by the corresponding change in x.
- A line through (x_1,y_1) with gradient m has $y-y_1=m(x-x_1)$.
- Watch for: A vertical line has no finite gradient; do not force it into y=mx+c.

3.2 · Arithmetic sequences and nth terms · Core

Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
- This lesson studies **common difference** 公差: The constant added between consecutive terms of an arithmetic sequence.

The method

- Find a constant difference for an arithmetic sequence.
- Its nth term is a+(n-1)d.
- A term-to-term rule describes how to reach the next term; a position-to-term rule gives a term directly.

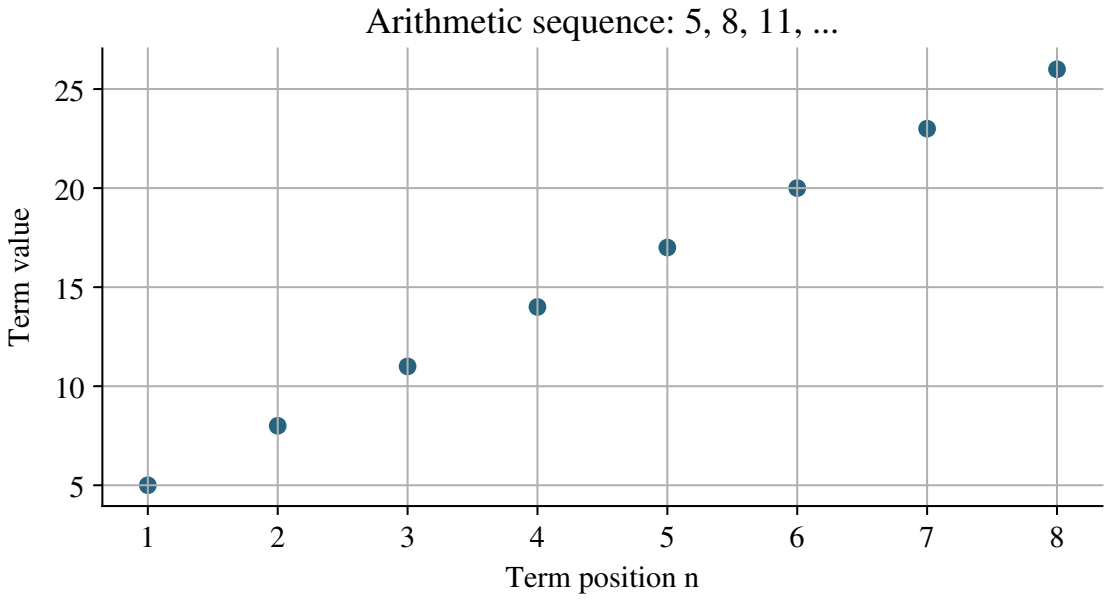
A worked case

$$u_n = a + (n-1)d$$

■ **Worked example**

For 5,8,11,14,... the common difference is 3. The nth term is 5+3(n-1)=3n+2. At n=8, $u_8=26$. To find the position of 62, solve 3n+2=62, giving n=20.

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Where it goes wrong

- The first term has index 1, so the exponent is n-1.
- A sequence is a list; a series is a sum.
- A geometric sequence can alternate in sign and still converge.

> ■ **Watch out**
>
> **Not true:** The first term of a sequence has index zero. A sum to infinity exists only when $-1<r<1$; for $r=2$ the terms grow without limit.

Applying it

- Generate several terms and check a proposed nth-term rule.
- Infinite geometric series and advanced sum formulae are excluded from this Foundation/Core lesson.

In your course

- OxfordAQA International GCSE Mathematics (9260), Core tier, specification section 3.2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ Key idea

- **Common difference**: The constant added between consecutive terms of an arithmetic sequence.
- Find a constant difference for an arithmetic sequence.
- Watch for: the first term has index 1, so the exponent is n-1.

3.2 · Sequences, series and recurrence · Extension

Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
- This lesson studies **common ratio** 公比: The constant multiplier between consecutive terms of a geometric sequence.

The method

- For an arithmetic progression, u_n=a+(n-1)d and S_n=n(2a+(n-1)d)/2.
- For a geometric progression, u_n=ar^(n-1) and S_n=a(1-r^n)/(1-r).
- An infinite geometric sum exists only if |r|<1.

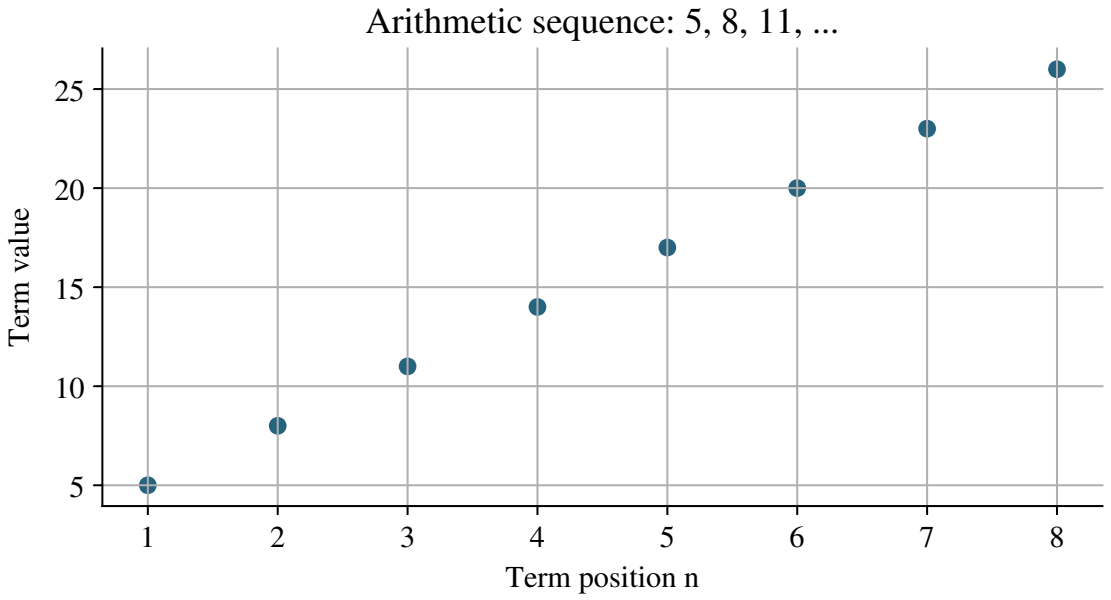
A worked case

$$u_n = a + (n-1)d, \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

■ Worked example

For a=5,d=3,n=8, u_8=5+7×3=26 and S_8=8(10+21)/2=124. For a=12,r=1/2, S infinity=12/(1-1/2)=24. For u_(n+1)=2u_n+1 with u_1=1, the next terms are 3,7,15.

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Where it goes wrong

- The first term has index 1, so the exponent is n-1.
- A sequence is a list; a series is a sum.
- A geometric sequence can alternate in sign and still converge.

> ■ **Watch out**
>
> **Not true:** Every geometric series has a finite sum to infinity. A sum to infinity exists only when $-1<r<1$; for $r=2$ the terms grow without limit.

Applying it

- Explain whether the context justifies additive or multiplicative change.
- In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.

In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Common ratio**: The constant multiplier between consecutive terms of a geometric sequence.
- For an arithmetic progression, u_n=a+(n-1)d and S_n=n(2a+(n-1)d)/2.
- Watch for: the first term has index 1, so the exponent is n-1.

3.2 · Derivatives and stationary points · Extension

What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative** 导数: The instantaneous rate of change, also the gradient of a tangent.

The method

- For a polynomial term ax^n with nonnegative integer n, the gradient term is anx^(n-1).
- Add the differentiated terms.
- A stationary point has zero gradient.

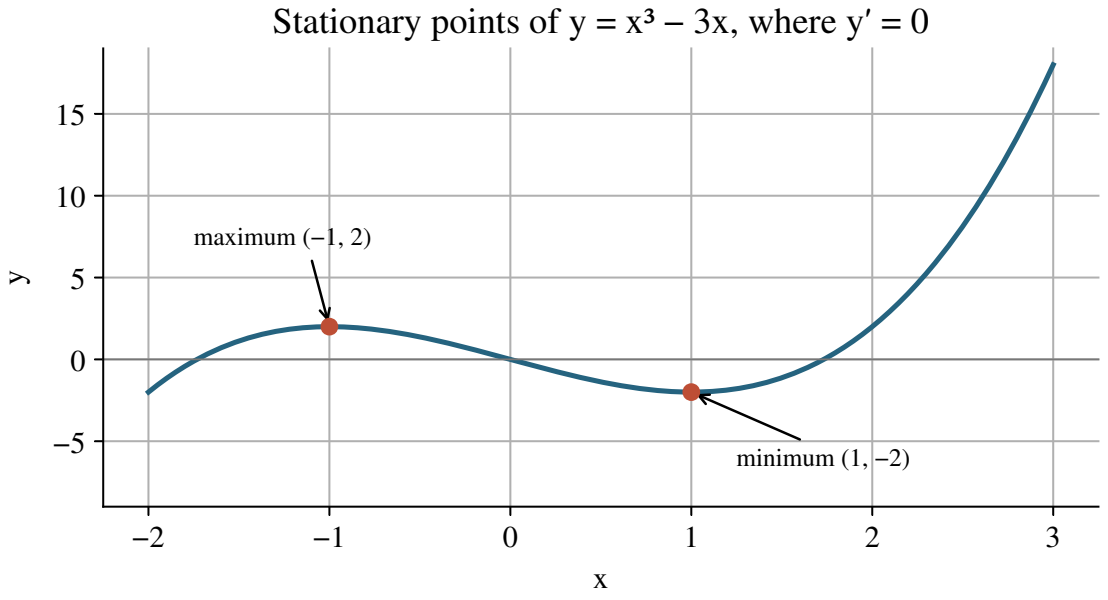
A worked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \qquad f'(x) = 0$$

■ **Worked example**

For $y=x^3-3x$, $dy/dx=3x^2-3$. At x=1, the gradient is 0 and y=-2. The second derivative is 6x, positive at x=1, so this is a local minimum. At x=-1, y=2 and the second derivative is negative, giving a local maximum.

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Where it goes wrong

- A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.
- An endpoint can also produce an extreme value on a restricted domain.

> ■ **Watch out**
>
> **Not true:** Every point with zero derivative is a local maximum or minimum. A zero derivative can also be a point of inflection: $y=x^3$ at $x=0$ is neither a maximum nor a minimum.

Applying it

- This advanced-tier IGCSE lesson is limited to polynomial differentiation, tangent gradients and stationary points.
- Chain, product, quotient and implicit differentiation are excluded.

In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.2.

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- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ **Key idea**

- **Derivative**: The instantaneous rate of change, also the gradient of a tangent.
- For a polynomial term ax^n with nonnegative integer n, the gradient term is anx^(n-1).
- Watch for: A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.

3.3 · Angles, lengths and area · Core

Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor** 相似比: The multiplier that relates corresponding lengths in similar shapes.

The method

- Use angle facts with a stated reason.
- Similar shapes have equal corresponding angles and proportional corresponding lengths.
- Areas of rectangles and triangles come from their dimensions; compound shapes can be split into simpler parts.

A worked case

$$A_{\text{rectangle}} = LW, \quad A_{\text{triangle}} = \frac{1}{2}bh$$

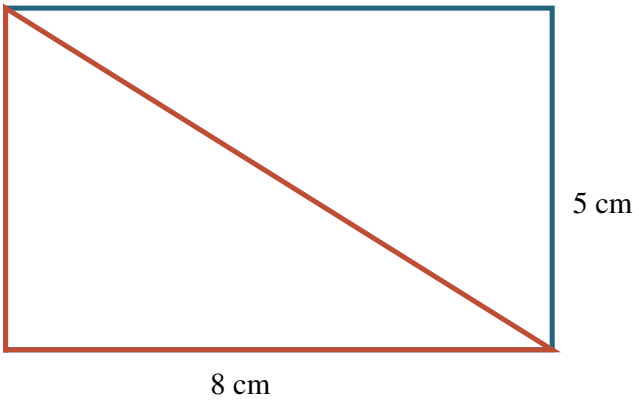
■ **Worked example**

A rectangle of length 8 cm and width 5 cm has area A=LW=40 cm^2. A triangle on the same base and height has area A=bh/2=20 cm^2. For a pentagon, the interior-angle sum is (5-2)×180=540°.

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Same base and height, half the area

Rectangle: 40 cm²; triangle: 20 cm²



Where it goes wrong

- Equal angles alone establish similarity, not equal size.
- Use corresponding lengths in the same order.
- Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

■ Watch out

Not true: A triangle and rectangle with the same base and height have the same area. Volume scales by the cube of the length factor: doubling every length multiplies the volume by 8.

Applying it

- Use a labelled sketch and appropriate units.
- This Foundation/Core lesson does not test area/volume scale factors or advanced circle-theorem proofs.

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In your course

- OxfordAQA International GCSE Mathematics (9260), Core tier, specification section 3.3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Scale factor**: The multiplier that relates corresponding lengths in similar shapes.
- Use angle facts with a stated reason.
- Watch for: equal angles alone establish similarity, not equal size.

3.3 · Angle reasoning, similarity and mensuration · Extension

Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor** 相似比: The multiplier that relates corresponding lengths in similar shapes.

The method

- For similar shapes with length scale factor k, areas scale by k^2 and volumes by k^3.
- State angle reasons explicitly.
- A circle's tangent is perpendicular to the radius at the contact point.

A worked case

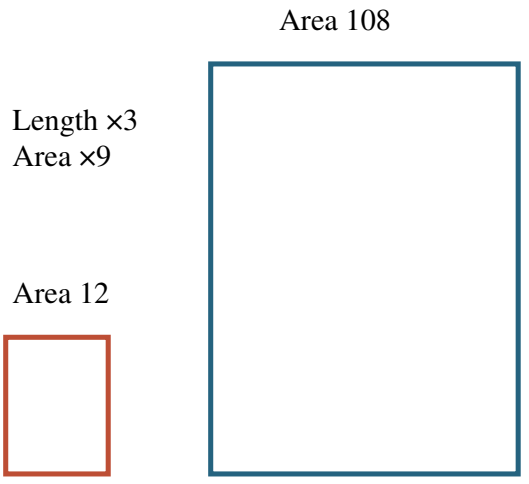
$$\frac{A_2}{A_1} = k^2, \qquad \frac{V_2}{V_1} = k^3$$

■ Worked example

If model-to-real length factor is 3, a model area of 12 cm^2 gives $12\times3^2=108$ cm^2 and a model volume of 8 cm^3 gives $8\times3^3=216$ cm^3. A cylinder with r=3,h=5 has volume $\pi r^2 h=45\pi$.

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A length scale factor is squared for area



Where it goes wrong

- Equal angles alone establish similarity, not equal size.
- Use corresponding lengths in the same order.
- Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

> ■ **Watch out**
>
> **Not true:** Doubling every length of a solid doubles its volume. Volume scales by the cube of the length factor: doubling every length multiplies the volume by 8.

Applying it

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion.
- A scale drawing is evidence only when the task permits measurement.

In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.3.

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- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Scale factor**: The multiplier that relates corresponding lengths in similar shapes.
- For similar shapes with length scale factor k, areas scale by k^2 and volumes by k^3.
- Watch for: equal angles alone establish similarity, not equal size.

3.3 · Right triangles and non-right triangles · Core

Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
- This lesson studies **hypotenuse** 斜边: The side opposite the right angle in a right-angled triangle.

The method

- Use Pythagoras in a right triangle and use sine, cosine or tangent with the sides labelled relative to the chosen angle.

A worked case

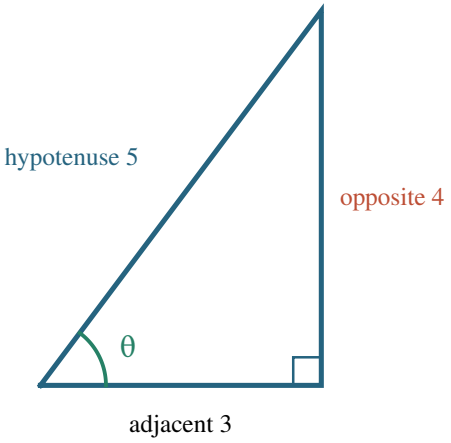
$$a^2 + b^2 = c^2, \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Worked example

The ladder length is $c=\sqrt{(3^2+4^2)}=5$ m. Its angle to the ground satisfies $\tan\theta=4/3$, so $\theta\approx53.1°$. A right triangle with legs 6 and 8 has area $6\times8/2=24$.

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Label the known sides before choosing a rule



opposite: across from θ
adjacent: next to θ
hypotenuse: across from the right angle

tan θ = 4/3 sin θ = 4/5
cos θ = 3/5

Where it goes wrong

- Label sides relative to the chosen angle.
- Pythagoras needs a right angle.
- A calculator angle mode error can produce a plausible but wrong result.
- Keep unrounded values for later steps.

Watch out

Not true: Pythagoras applies to every triangle, including triangles without a right angle. Pythagoras needs a right angle; for other triangles use the cosine rule.

Applying it

- This Foundation/Core lesson uses right-angled triangles only.
- Sine and cosine rules for non-right triangles belong to the advanced-tier lesson.

In your course

- OxfordAQA International GCSE Mathematics (9260), Core tier, specification section 3.3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ **Key idea**

- **Hypotenuse**: The side opposite the right angle in a right-angled triangle.
- Use Pythagoras in a right triangle and use sine, cosine or tangent with the sides labelled relative to the chosen angle.
- Watch for: label sides relative to the chosen angle.

3.3 · Right triangles and non-right triangles · Extension

Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
- This lesson studies **hypotenuse** 斜边: The side opposite the right angle in a right-angled triangle.

The method

- In a right triangle $a^2+b^2=c^2$; $\sin\theta$=opposite/hypotenuse, $\cos\theta$=adjacent/hypotenuse and $\tan\theta$=opposite/adjacent.
- For other triangles, use the sine or cosine rule, or area=ab sin C/2.

A worked case

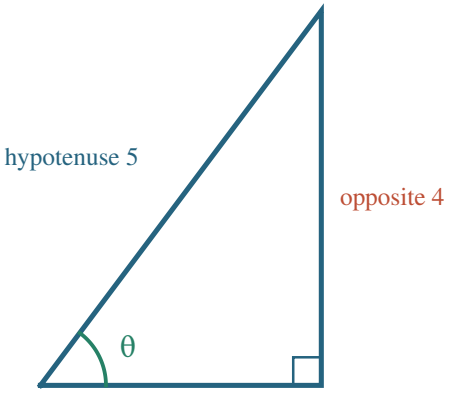
$$a^2 + b^2 = c^2, \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

■ **Worked example**

The ladder length is $c=\sqrt{(3^2+4^2)}=5$ m. Its angle to the ground satisfies $\tan\theta=4/3$, so $\theta\approx53.1°$. With two sides 6 and 8 enclosing 60°, $c^2=6^2+8^2-2\times6\times8\cos60°=52$.

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Label the known sides before choosing a rule



opposite: across from θ
adjacent: next to θ
hypotenuse: across from the right angle

tan θ = 4/3 sin θ = 4/5
cos θ = 3/5

Where it goes wrong

- Label sides relative to the chosen angle.
- Pythagoras needs a right angle.
- A calculator angle mode error can produce a plausible but wrong result.
- Keep unrounded values for later steps.

Watch out

Not true: Pythagoras applies to every triangle, including triangles without a right angle. Pythagoras needs a right angle; for other triangles use the cosine rule.

Applying it

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule.
- Explain why the chosen triangle contains the required length or angle.

In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.3.

- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Hypotenuse**: The side opposite the right angle in a right-angled triangle.
- In a right triangle $a^2+b^2=c^2$; $\sin\theta$=opposite/hypotenuse, $\cos\theta$=adjacent/hypotenuse and $\tan\theta$=opposite/adjacent.
- Watch for: label sides relative to the chosen angle.

3.3 · Vectors and transformation geometry · Core

Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
- This lesson studies **resultant** 合向量: The vector sum representing the combined displacement or force.

The method

- Add corresponding vector components and subtract position vectors to find a displacement.
- A translation moves every point by the same vector; a scalar multiple changes length and possibly direction.

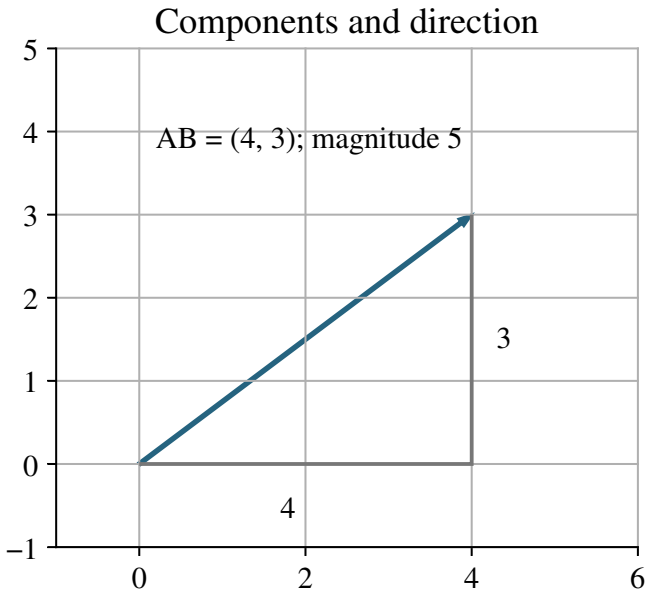
A worked case

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \qquad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

■ Worked example

With a=(4,1) and b=(1,3), a+b=(5,4). From A=(1,2) to B=(5,5), displacement AB=(4,3). Its magnitude is $\sqrt{(4^2+3^2)}=5$.

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Where it goes wrong

- The order of subtraction matters: BA=-AB.
- Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient.
- Negative enlargement reverses position about its centre.

> ■ **Watch out**
>
> **Not true:** AB and BA have the same components. BA goes the opposite way to AB, so BA=−AB: every component changes sign.

Applying it

- Draw arrows with direction and identify the starting and ending points.
- Scalar products, spatial line equations and advanced angle calculations are excluded from this Foundation/Core lesson.

In your course

- OxfordAQA International GCSE Mathematics (9260), Core tier, specification section 3.3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Resultant**: The vector sum representing the combined displacement or force.
- Add corresponding vector components and subtract position vectors to find a displacement.
- Watch for: the order of subtraction matters: BA=-AB.

3.3 · Vectors and transformation geometry · Extension

Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
- This lesson studies **resultant** 合向量: The vector sum representing the combined displacement or force.

The method

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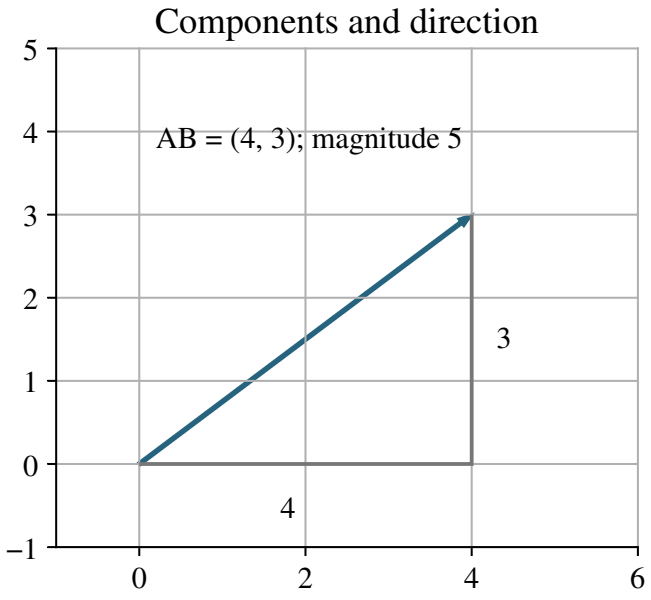
A worked case

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \qquad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

■ **Worked example**

With a=(4,1) and b=(1,3), a+b=(5,4). From A=(1,2) to B=(5,5), displacement AB=(4,3). Its magnitude is $\sqrt{(4^2+3^2)}=5$.

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>
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Applying it

- Draw arrows with direction and identify the starting and ending points.
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In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Resultant**: The vector sum representing the combined displacement or force.
- Add corresponding vector components and subtract position vectors to find a displacement.
- Watch for: the order of subtraction matters: BA=-AB.

3.3 · Reflections, rotations and enlargements · Core

Where is the enlargement centre?

- A logo is enlarged around a point away from the origin. Multiplying its coordinates alone puts it in the wrong place.
- This lesson studies **centre of enlargement** 位似中心: The point from which each point's displacement is multiplied by a scale factor.

The method

- A translation adds a vector.
- A reflection reverses signed perpendicular distance from a mirror line.
- A rotation needs a centre, angle and direction.
- For enlargement from C, use new P=C+k(P-C).

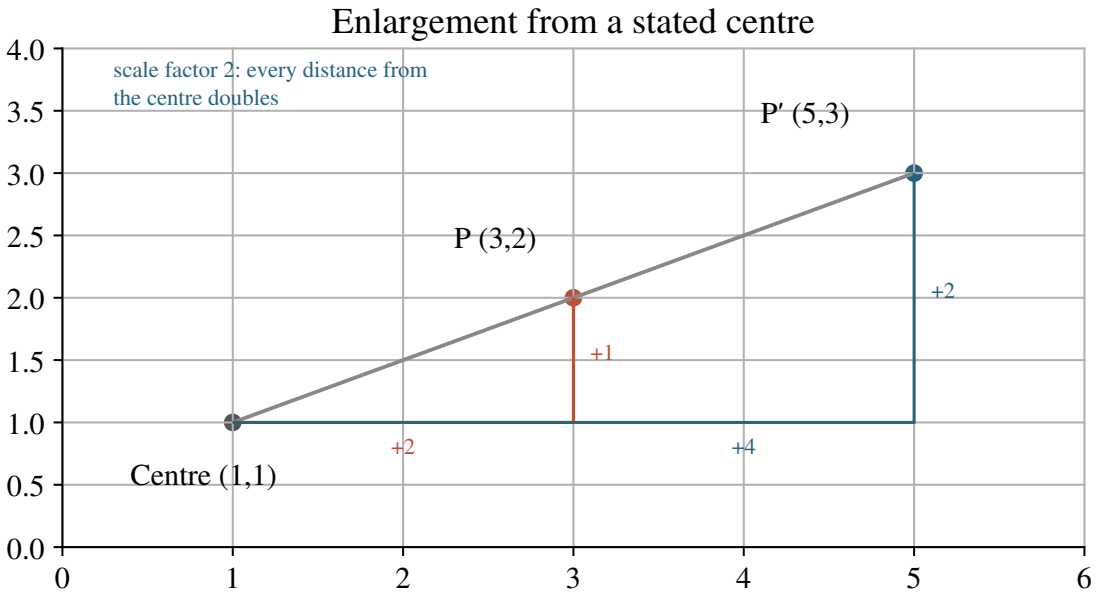
A worked case

$$\mathbf{p}_{\text{image}} = \mathbf{c} + k(\mathbf{p} - \mathbf{c})$$

■ **Worked example**

For C=(1,1),P=(3,2),k=2, P-C=(2,1), so new P=(1,1)+2(2,1)=(5,3). Reflection of (3,2) in the y-axis gives (-3,2). A 90° anticlockwise rotation about the origin gives (-2,3).

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Where it goes wrong

- A rotation needs its centre and direction, not just an angle.
- A negative enlargement factor places the image on the opposite side of the centre.
- A translation does not change orientation or size.

■ Watch out

Not true: Every enlargement is centred at the origin unless its scale factor is negative. An enlargement can have any centre; a negative scale factor puts the image on the other side of the centre.

Applying it

- Describe a transformation completely before constructing the image.
- Check corresponding distances and angles.
- For combined transformations, apply them in the stated order; they usually do not commute.

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In your course

- OxfordAQA International GCSE Mathematics (9260), Core tier, specification section 3.3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Centre of enlargement**: The point from which each point's displacement is multiplied by a scale factor.
- A translation adds a vector.
- Watch for: A rotation needs its centre and direction, not just an angle.

3.3 · Reflections, rotations and enlargements · Extension

Where is the enlargement centre?

- A logo is enlarged around a point away from the origin. Multiplying its coordinates alone puts it in the wrong place.
- This lesson studies **centre of enlargement** 位似中心: The point from which each point's displacement is multiplied by a scale factor.

The method

- A translation adds a vector.
- A reflection reverses signed perpendicular distance from a mirror line.
- A rotation needs a centre, angle and direction.
- For enlargement from C, use new P=C+k(P-C).

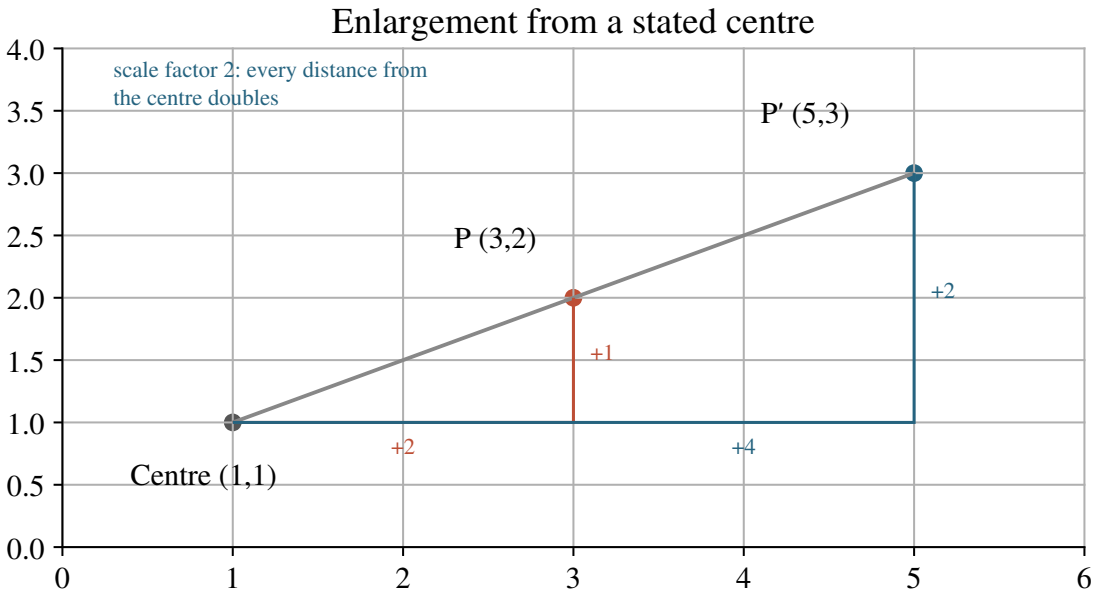
A worked case

$$\mathbf{p}_{\text{image}} = \mathbf{c} + k(\mathbf{p} - \mathbf{c})$$

Worked example

For C=(1,1),P=(3,2),k=2, P-C=(2,1), so new P=(1,1)+2(2,1)=(5,3). Reflection of (3,2) in the y-axis gives (-3,2). A 90° anticlockwise rotation about the origin gives (-2,3).

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Where it goes wrong

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- A negative enlargement factor places the image on the opposite side of the centre.
- A translation does not change orientation or size.

■ Watch out

Not true: Every enlargement is centred at the origin unless its scale factor is negative. An enlargement can have any centre; a negative scale factor puts the image on the other side of the centre.

Applying it

- Describe a transformation completely before constructing the image.
- Check corresponding distances and angles.
- For combined transformations, apply them in the stated order; they usually do not commute.

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In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Centre of enlargement**: The point from which each point's displacement is multiplied by a scale factor.
- A translation adds a vector.
- Watch for: A rotation needs its centre and direction, not just an angle.

3.3 · Constructions, loci and geometric conditions · Core

Where can both conditions hold?

- A router must be equally far from two rooms and within reach of a power point. Each condition creates a different set of possible positions.
- This lesson studies **locus** 轨迹: The set of all points satisfying a stated geometric condition.

The method

- Points equally distant from A and B lie on the perpendicular bisector of AB.
- Points at fixed distance r from C lie on a circle.
- Points equally distant from two intersecting lines lie on their angle bisectors.

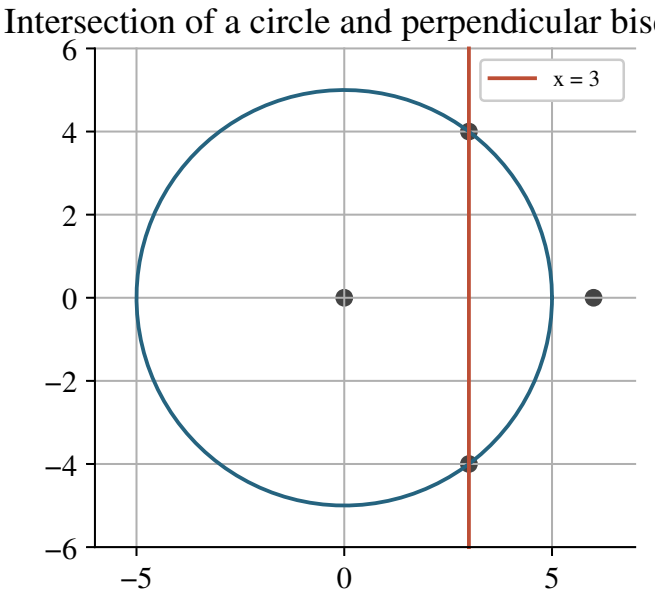
A worked case

$$x = 3, \qquad x^2 + y^2 = 25$$

■ Worked example

For A=(0,0) and B=(6,0), the perpendicular bisector is x=3. Points also 5 units from A satisfy $x^2+y^2=25$. Substituting x=3 gives $y^2=16$, so (3,4) and (3,-4) satisfy both conditions.

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Where it goes wrong

- The perpendicular bisector concerns distance to two points; the angle bisector concerns distance to two lines.
- A sketch is not a ruler-and-compass construction: preserve arcs as evidence of the method.

> ■ **Watch out**
>
> **Not true:** Points equally distant from two points lie on their angle bisector. Points equidistant from two points lie on the perpendicular bisector of the segment joining them; the angle bisector is equidistant from two lines.

Applying it

- Translate each condition into a locus before finding intersections.
- For a region closer to A than B, choose the correct side of the perpendicular bisector and show whether a boundary is allowed.

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In your course

- OxfordAQA International GCSE Mathematics (9260), Core tier, specification section 3.3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Locus**: The set of all points satisfying a stated geometric condition.
- Points equally distant from A and B lie on the perpendicular bisector of AB.
- Watch for: the perpendicular bisector concerns distance to two points; the angle bisector concerns distance to two lines.

3.3 · Constructions, loci and geometric conditions · Extension

Where can both conditions hold?

- A router must be equally far from two rooms and within reach of a power point. Each condition creates a different set of possible positions.
- This lesson studies **locus** 轨迹: The set of all points satisfying a stated geometric condition.

The method

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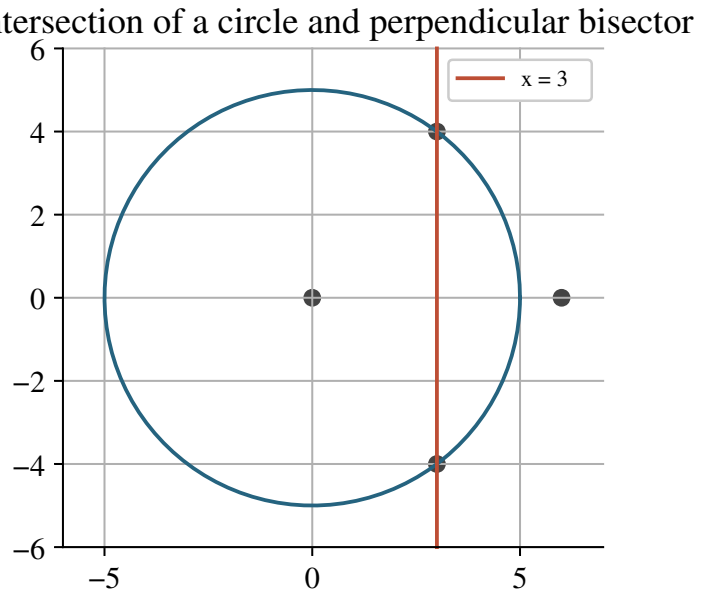
A worked case

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■ Worked example

For A=(0,0) and B=(6,0), the perpendicular bisector is x=3. Points also 5 units from A satisfy $x^2+y^2=25$. Substituting x=3 gives $y^2=16$, so (3,4) and (3,-4) satisfy both conditions.

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Where it goes wrong

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- A sketch is not a ruler-and-compass construction: preserve arcs as evidence of the method.

> ■ **Watch out**
>
> **Not true:** Points equally distant from two points lie on their angle bisector. Points equidistant from two points lie on the perpendicular bisector of the segment joining them; the angle bisector is equidistant from two lines.

Applying it

- Translate each condition into a locus before finding intersections.
- For a region closer to A than B, choose the correct side of the perpendicular bisector and show whether a boundary is allowed.

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In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Locus**: The set of all points satisfying a stated geometric condition.
- Points equally distant from A and B lie on the perpendicular bisector of AB.
- Watch for: the perpendicular bisector concerns distance to two points; the angle bisector concerns distance to two lines.

3.3 · Circle theorems and reasoned proofs · Extension

Which angles see the same chord?

- Two observers see the same chord from the circle. Their angles are linked by a theorem rather than by the apparent size of the drawing.
- This lesson studies **cyclic quadrilateral** 圆内接四边形: A quadrilateral whose four vertices lie on one circle.

The method

- The angle at the centre is twice the angle at the circumference on the same arc.
- Angles in the same segment are equal.
- Opposite angles of a cyclic quadrilateral sum to 180°.
- A radius is perpendicular to a tangent at contact.

A worked case

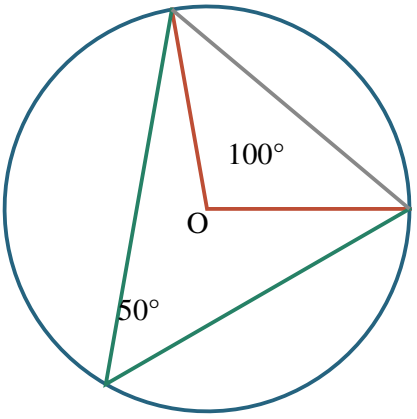
$$\theta_{\text{centre}} = 2\theta_{\text{circumference}}$$

■ Worked example

If a central angle is 100°, the corresponding angle at the circumference is 50°. In a cyclic quadrilateral with one angle 112°, its opposite angle is 180-112=68°. A radius meeting a tangent gives 90°, even if the drawing looks oblique.

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The centre angle is twice the circumference angle



Where it goes wrong

- Identify the same chord and the correct arc before using a theorem.
- Two visible right angles do not prove a quadrilateral cyclic without a valid converse argument.
- A diagram need not be to scale.

■ Watch out

Not true: Every quadrilateral has opposite angles summing to 180°. Opposite angles sum to 180° only in a cyclic quadrilateral, one whose vertices lie on a circle.

Applying it

- Write one reason alongside each angle calculation.
- For the alternate-segment theorem, name the tangent and chord, then identify the angle in the opposite segment.
- Use auxiliary radii only when they help the proof.

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In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Cyclic quadrilateral**: A quadrilateral whose four vertices lie on one circle.
- The angle at the centre is twice the angle at the circumference on the same arc.
- Watch for: identify the same chord and the correct arc before using a theorem.

3.3 · Plane matrix transformations · Extension

Where do the basis vectors go?

- A map stretches one direction and shears another. A matrix records the transformation of each basis vector.
- This lesson studies **transformation matrix** 变换矩阵: A matrix whose action on a column vector defines a plane transformation.

The method

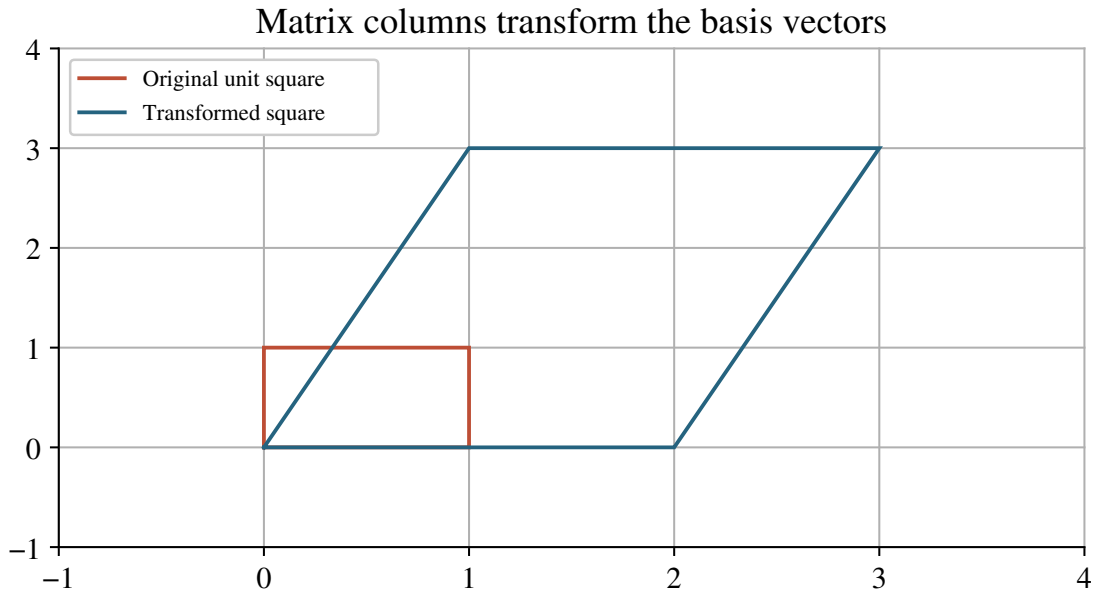
- Represent a plane transformation using a 2×2 matrix acting on column vectors.
- The columns give the images of the two basis vectors.
- Combine transformations in the specified order.

A worked case

$$\begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

■ Worked example

For A=[[2,1],[0,3]], A(1,2)=(4,6). A maps (1,0) to (2,0) and (0,1) to (1,3). Matrix multiplication records composition; the rightmost matrix acts first.



Where it goes wrong

- Matrix multiplication is usually not commutative.
- Treat coordinates as column vectors and apply a composition in the correct order.

> ■ **Watch out**
>
> **Not true:** The order of a pair of matrix transformations never changes the image. A square matrix with determinant 0 is singular and has no inverse.

Applying it

- This 9260 Extension lesson concerns plane transformation matrices.
- Eigenvalues, diagonalisation and characteristic equations are outside its scope.

In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Transformation matrix**: A matrix whose action on a column vector defines a plane transformation.
- Represent a plane transformation using a 2×2 matrix acting on column vectors.
- Watch for: matrix multiplication is usually not commutative.

3.3 · Integrals, area and accumulation · Extension

How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
- This lesson studies **antiderivative** 原函数: A function whose derivative equals the given integrand.

The method

- Integrate polynomial terms by increasing the power by one and dividing by the new power.
- Include a constant for an indefinite integral.
- Evaluate a definite integral by subtracting antiderivative values at the limits.

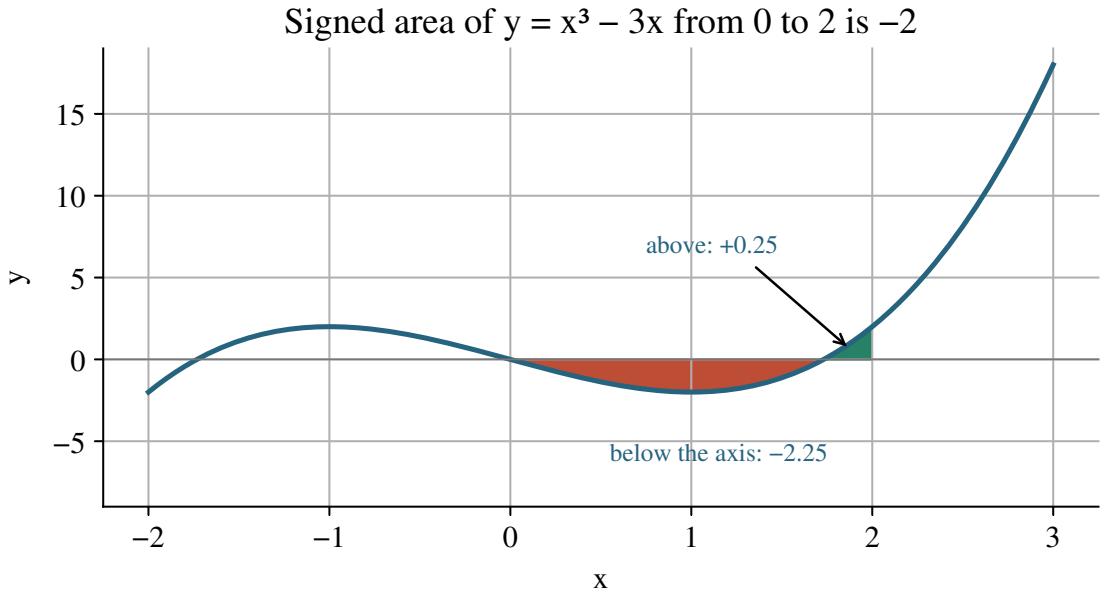
A worked case

$$\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

■ **Worked example**

For v(t)=3t^2-3 over 0≤t≤2, displacement=[t^3-3t]_0^2=2. Since v changes sign at t=1, distance=-[t^3-3t]_0^1+[t^3-3t]_1^2=2+4=6. The constants cancel only for a definite integral.

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Where it goes wrong

- Use the limits in the specified order.
- The constant cancels for a definite integral but is needed for an indefinite integral.
- Total positive area may require splitting at an axis crossing.

> ■ **Watch out**
>
> **Not true:** A definite integral equals the total positive area, even when the graph crosses the axis. Area below the axis counts as negative, so a definite integral can be smaller than the total area, or even zero.

Applying it

- This 9260 Extension lesson uses polynomial integration and area.
- Logarithmic integrals, substitution, parts and differential equations are outside its scope.

In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.3.

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- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Antiderivative**: A function whose derivative equals the given integrand.
- Integrate polynomial terms by increasing the power by one and dividing by the new power.
- Watch for: use the limits in the specified order.

3.4 · Centre, spread and data displays · Core

Can one average tell the whole story?

- Two groups have the same median but different spread. One summary cannot describe both location and consistency.
- This lesson studies **range** 极差: The maximum observed value minus the minimum observed value.

The method

- The mean is total divided by count.
- The median is the central value after sorting.
- The range is maximum minus minimum.
- Use frequency tables, bar charts and suitable comparisons.

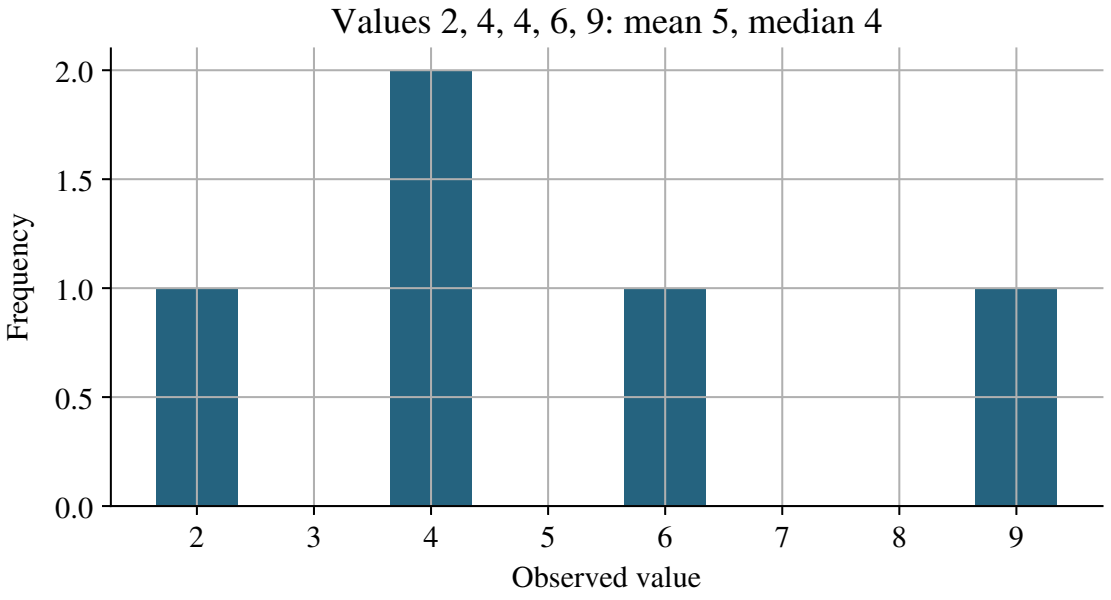
A worked case

$$\overline{x} = \frac{\sum x_i}{n}$$

Worked example

For 2,4,4,6,9, total=25 and count=5, so mean=5. The central value is 4, so median=4. The range is 9-2=7. Explain both a typical value and the spread.

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Where it goes wrong

- The tallest histogram bar need not contain the most observations.
- A grouped mean is an estimate.
- Correlation does not prove causation, and extrapolation extends beyond the observed range.

> **Watch out**
>
> **Not true:** The mean of a list must be one of its observed values. With unequal class widths the height is frequency density; frequency is the area of the bar.

Applying it

- A bar chart uses separate bars for categories.
- Unequal-class-width histograms and formal density calculations are outside this Foundation/Core lesson.

In your course

- OxfordAQA International GCSE Mathematics (9260), Core tier, specification section 3.4.

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- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Range**: The maximum observed value minus the minimum observed value.
- The mean is total divided by count.
- Watch for: the tallest histogram bar need not contain the most observations.

3.4 · Data summaries, histograms and interpretation · Extension

Can one average tell the whole story?

- Two groups have the same median but different spread. One summary cannot describe both location and consistency.
- This lesson studies **frequency density** 频率密度: Frequency divided by class width, used as histogram height.

The method

- Compare an appropriate average and spread in context.
- A histogram uses area for frequency, so height=frequency/class width.
- Grouped estimates assume representative values within intervals.

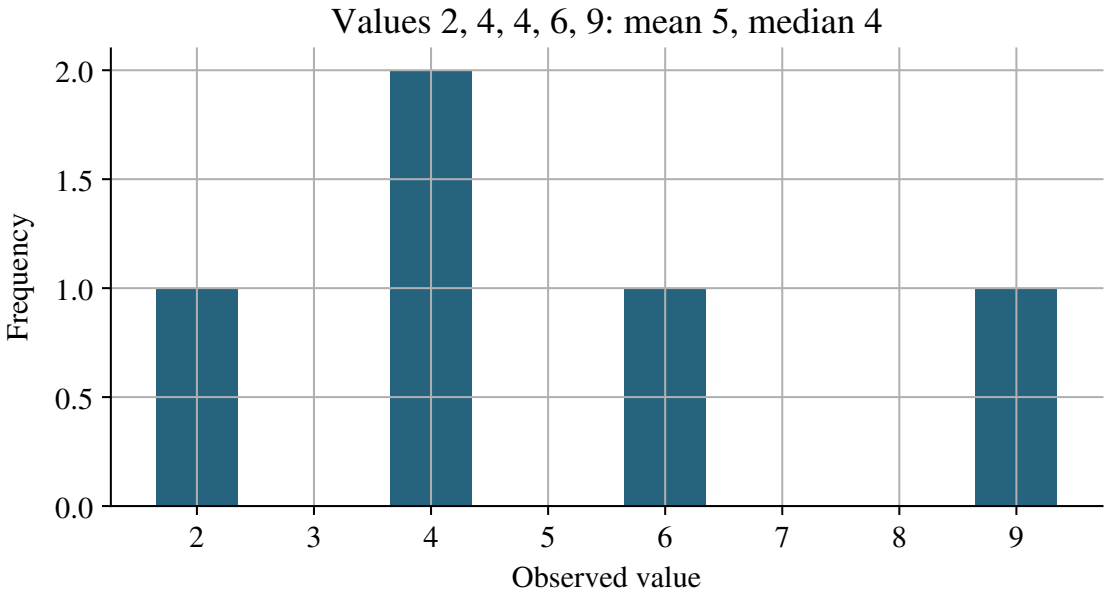
A worked case

$$\overline{x} = \frac{\sum x_i}{n}, \qquad \text{density} = \frac{\text{frequency}}{\text{class width}}$$

Worked example

A class from 10 to 20 with frequency 30 has density 30/10=3. A class from 20 to 40 with frequency 20 has density 20/20=1. Its wider bar must not be mistaken for a larger density. For values 2,4,4,6,9, the median is 4 and mean is 5.

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Where it goes wrong

- The tallest histogram bar need not contain the most observations.
- A grouped mean is an estimate.
- Correlation does not prove causation, and extrapolation extends beyond the observed range.

Watch out

Not true: A histogram bar's height equals its frequency, even with unequal class widths. With unequal class widths the height is frequency density; frequency is the area of the bar.

Applying it

- Choose a display that fits the data type.
- Give both a numerical comparison and what it means for the population; do not infer more precision than the sample supports.

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In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Frequency density**: Frequency divided by class width, used as histogram height.
- Compare an appropriate average and spread in context.
- Watch for: the tallest histogram bar need not contain the most observations.

3.4 · Probability trees and outcomes · Core

What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** 条件概率: The probability of an event after restricting the sample space to a stated condition.

The method

- A probability lies between 0 and 1.
- Exhaustive, mutually exclusive outcomes have probabilities summing to 1.
- Multiply successive branch probabilities and add separate routes to an outcome.

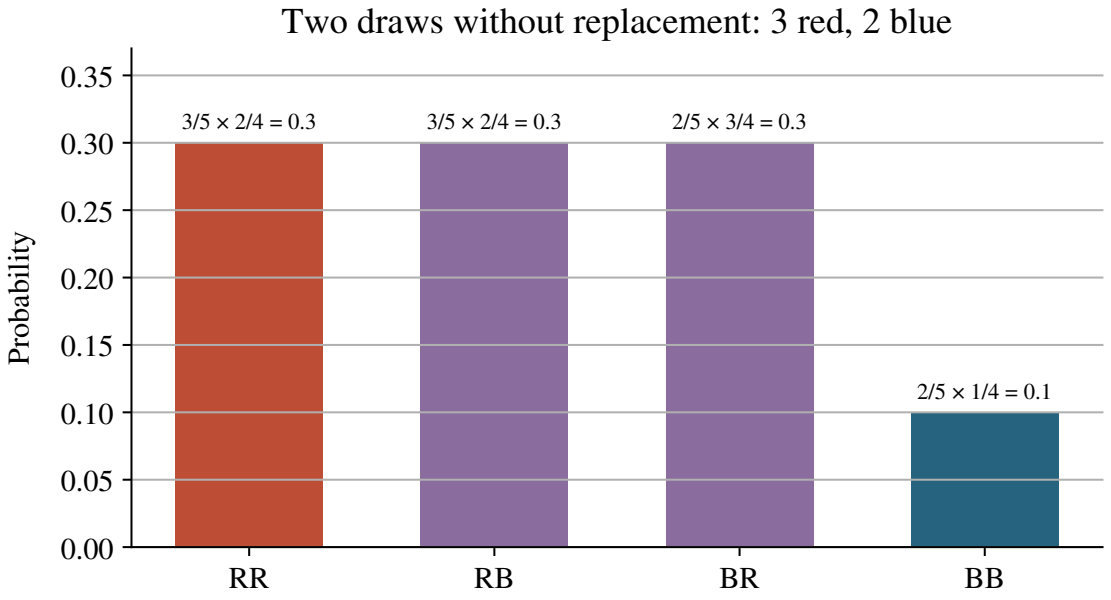
A worked case

$$P(RR) = P(R_1)P(R_2 \mid R_1)$$

■ Worked example

A bag contains 3 red and 2 blue counters. With replacement, P(two red) = $3/5 \times 3/5 = 9/25 = 0.36$. Without replacement, the red-red branch is $3/5 \times 2/4 = 0.3$. Label each branch before multiplying.

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Where it goes wrong

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.
- Two disjoint events with positive probability are not independent.

> ■ **Watch out**
>
> **Not true:** Mutually exclusive events with positive probabilities must be independent. Mutually exclusive events with positive probabilities are dependent: if one happens, the other cannot, so $P(A \text{ and } B)=0 \neq P(A)P(B)$.

Applying it

- Use a frequency table or a simple tree before calculating.
- Formal conditional probability formulae are outside this Foundation/Core support lesson.

In your course

- OxfordAQA International GCSE Mathematics (9260), Core tier, specification section 3.4.

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- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Conditional probability**: The probability of an event after restricting the sample space to a stated condition.
- A probability lies between 0 and 1.
- Watch for: mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.

3.4 · Probability, trees and conditional reasoning · Extension

What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** 条件概率: The probability of an event after restricting the sample space to a stated condition.

The method

- Multiply along a tree branch and add disjoint branches.
- With replacement, the composition stays fixed.
- Conditional probability is P(A given B)=P(A∩B)/P(B), for P(B)>0.

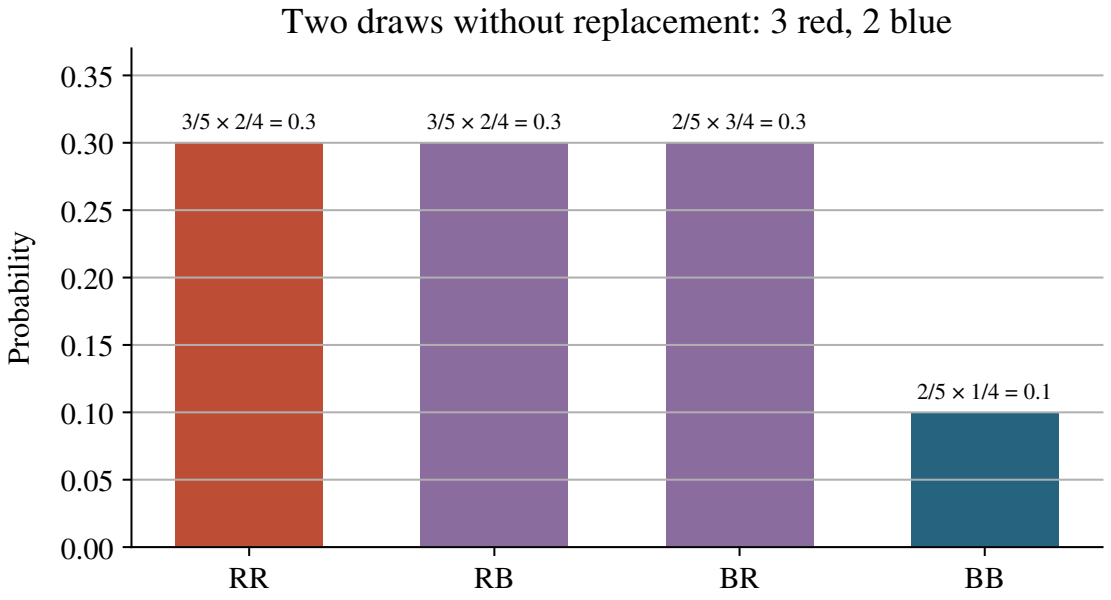
A worked case

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \qquad P(B) > 0$$

Worked example

Without replacement, P(two red)=3/5×2/4=3/10. P(one of each)=3/5×2/4+2/5×3/4=3/5. If P(A∩B)=0.12 and P(B)=0.3, P(A given B)=0.4.

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Where it goes wrong

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.
- Two disjoint events with positive probability are not independent.

> ■ **Watch out**
>
> **Not true:** Mutually exclusive events with positive probabilities must be independent. Mutually exclusive events with positive probabilities are dependent: if one happens, the other cannot, so $P(A \text{ and } B)=0 \neq P(A)P(B)$.

Applying it

- A two-way table makes the restricted denominator visible.
- Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.

In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.4.

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- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ **Key idea**

- **Conditional probability**: The probability of an event after restricting the sample space to a stated condition.
- Multiply along a tree branch and add disjoint branches.
- Watch for: mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.

3.4 · Cumulative frequency and box plots · Extension

Which route is more consistent?

- Two bus routes have similar typical times but different reliability. Quartiles show the middle half of the journeys.
- This lesson studies **interquartile range** 四分位距: The difference between the upper and lower quartiles.

The method

- A cumulative frequency counts observations below successive class boundaries.
- Read quartiles at one quarter, one half and three quarters of the total frequency.
- A box plot represents minimum, lower quartile, median, upper quartile and maximum.

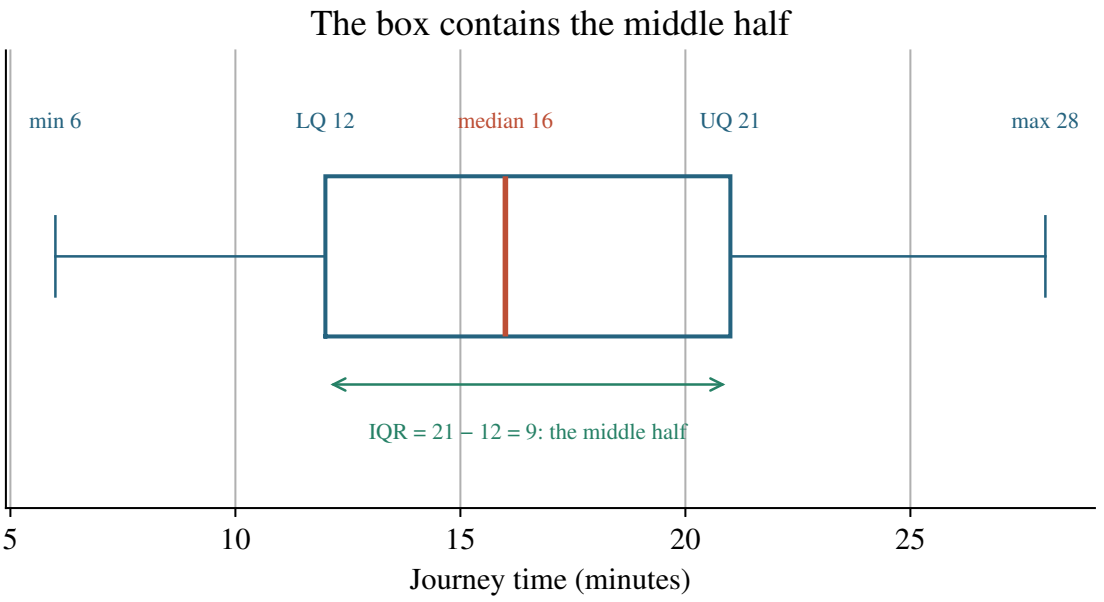
A worked case

$$\text{IQR} = Q_3 - Q_1$$

■ **Worked example**

For 80 observations, read Q1 at cumulative frequency 20, median at 40 and Q3 at 60. If Q1=12,Q3=21, then IQR=9. Compare the medians for typical journey time and the IQRs for consistency.

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Where it goes wrong

- Plot against class boundaries rather than midpoints.
- Grouped quartiles are estimates.
- The range is sensitive to extremes; the IQR describes only the middle half.

> ■ **Watch out**
>
> **Not true:** The interquartile range equals the maximum minus the minimum. The interquartile range is Q3−Q1; maximum minus minimum is the range.

Applying it

- Explain a comparison in the context of the measured quantity.
- An outlier rule may use Q1-1.5IQR and Q3+1.5IQR; use the rule specified in the task rather than assuming every graph follows it.

In your course

- OxfordAQA International GCSE Mathematics (9260), Extension tier, specification section 3.4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Interquartile range**: The difference between the upper and lower quartiles.
- A cumulative frequency counts observations below successive class boundaries.
- Watch for: plot against class boundaries rather than midpoints.

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